Computer algebra independent integration tests

4-Trig-functions/4.2-Cosine/4.2.1.3-g-tan-^p-a+b-cos-^m

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [22]. This is test number [88].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (22)	% 0.00 (0)
Mathematica	% 100.00 (22)	% 0.00 (0)
Maple	% 100.00 (22)	% 0.00 (0)
Maxima	% 77.27 (17)	% 22.73 (5)
Fricas	% 95.45 (21)	% 4.55 (1)
Sympy	% 4.55 (1)	% 95.45 (21)
Giac	% 90.91 (20)	% 9.09 (2)
Mupad	% 81.82 (18)	% 18.18 (4)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

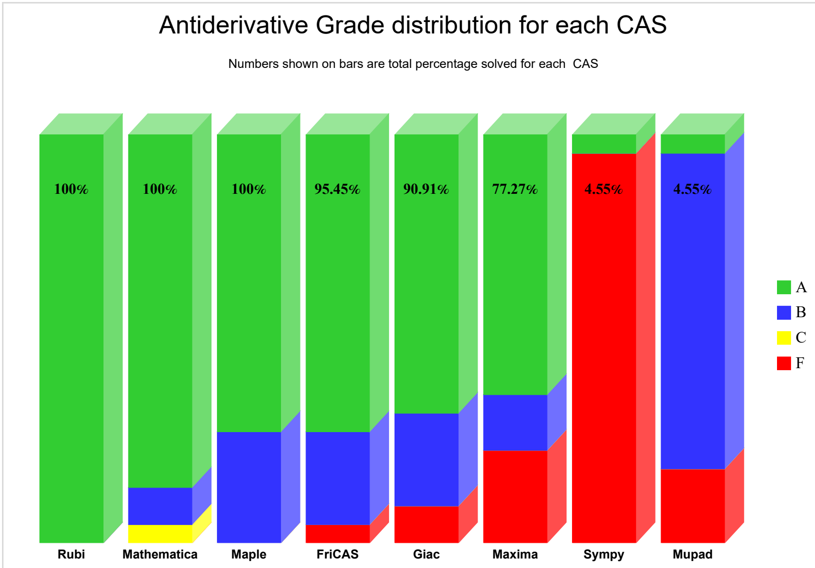
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

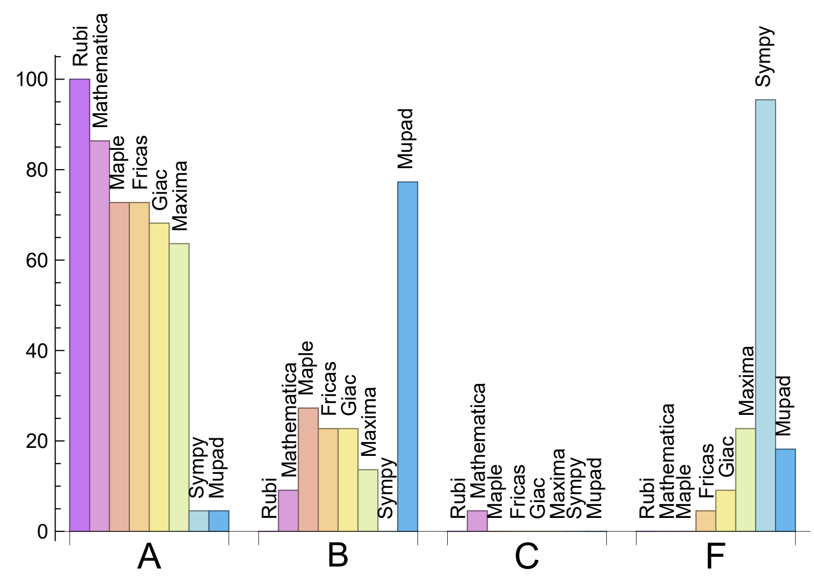
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	86.36	9.09	4.55	0.00
Maple	72.73	27.27	0.00	0.00
Maxima	63.64	13.64	0.00	22.73
Fricas	72.73	22.73	0.00	4.55
Sympy	4.55	0.00	0.00	95.45
Giac	68.18	22.73	0.00	9.09
Mupad	4.55	77.27	0.00	18.18

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F**(**-1**).

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Maxima	5	20.00 %	0.00 %	80.00 %
Fricas	1	0.00 %	100.00 %	0.00 %
Sympy	21	100.00 %	0.00 %	0.00 %
Giac	2	50.00 %	0.00 %	50.00 %
Mupad	4	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

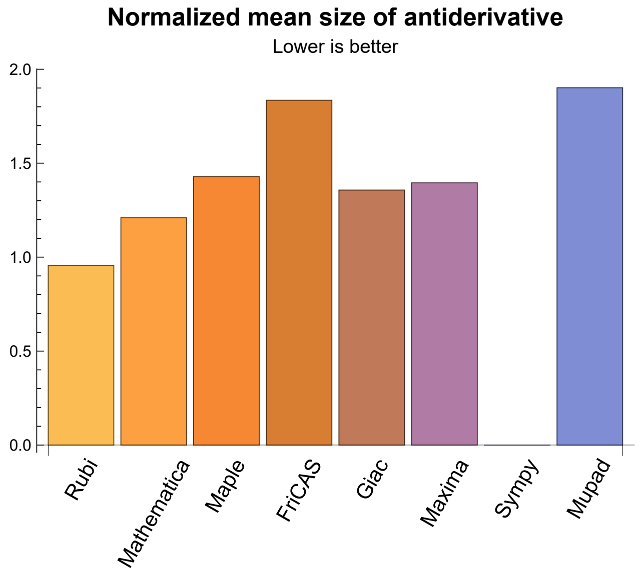
1.3 Performance

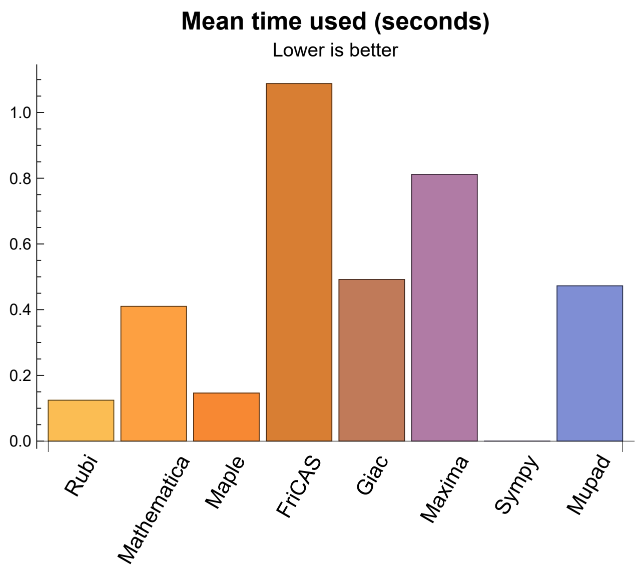
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.12	54.05	0.95	38.50	1.00
Mathematica	0.41	69.45	1.21	45.00	1.00
Maple	0.15	91.32	1.43	52.50	1.08
Maxima	0.81	48.18	1.40	46.00	1.22
Fricas	1.09	104.24	1.83	53.00	1.52
Sympy	0.00	0.00	0.00	0.00	0.00
Giac	0.49	69.70	1.36	52.00	1.17
Mupad	0.47	144.50	1.90	45.50	1.15

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{22}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {21}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://`

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

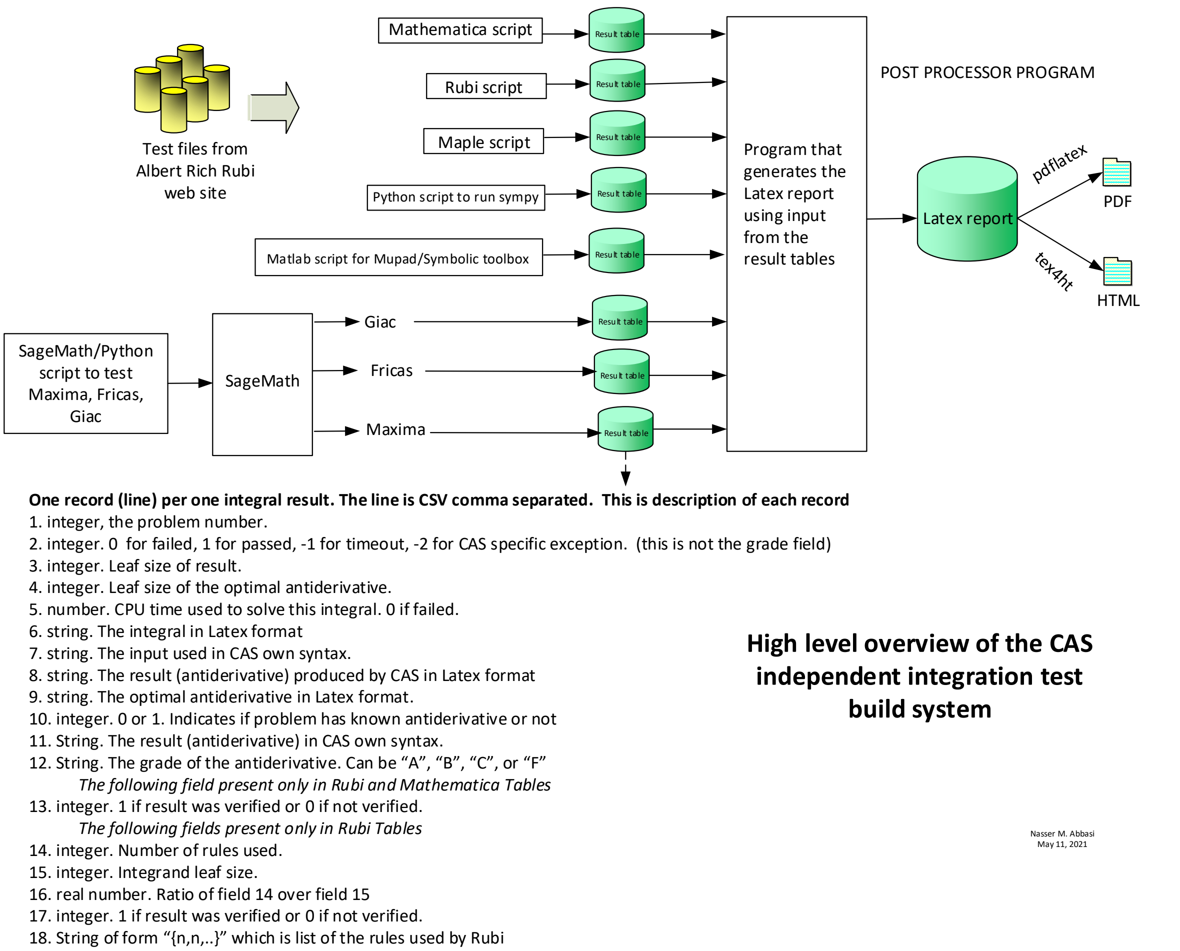
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22 }

B grade: { 1, 3 }

C grade: { 21 }

F grade: { }

2.1.3 Maple

A grade: { 2, 4, 5, 6, 7, 8, 9, 11, 13, 14, 15, 16, 17, 19, 20, 22 }

B grade: { 1, 3, 10, 12, 18, 21 }

C grade: { }

F grade: { }

2.1.4 Maxima

A grade: { 2, 4, 5, 6, 7, 9, 11, 13, 14, 16, 18, 19, 20, 22 }

B grade: { 1, 3, 8 }

C grade: { }

F grade: { 10, 12, 15, 17, 21 }

2.1.5 FriCAS

A grade: { 1, 2, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 17, 19, 22 }

B grade: { 3, 7, 16, 18, 20 }

C grade: { }

F grade: { 21 }

2.1.6 Sympy

A grade: { 22 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21 }

2.1.7 Giac

A grade: { 2, 4, 5, 6, 7, 8, 9, 11, 13, 14, 15, 16, 17, 19, 20 }

B grade: { 1, 3, 10, 12, 18 }

C grade: { }

F grade: { 21, 22 }

2.1.8 Mupad

A grade: { 22 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17 }

C grade: { }

F grade: { 18, 19, 20, 21 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	105	103	115	50	0	65	46
normalized size	1	1.00	3.18	3.12	3.48	1.52	0.00	1.97	1.39
time (sec)	N/A	0.081	0.133	0.069	0.297	0.793	0.000	0.707	0.458

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	18	15	15	0	15	13
normalized size	1	1.00	0.89	0.95	0.79	0.79	0.00	0.79	0.68
time (sec)	N/A	0.060	0.022	0.051	0.661	0.736	0.000	0.364	0.335

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	39	51	61	33	0	45	30
normalized size	1	1.00	2.60	3.40	4.07	2.20	0.00	3.00	2.00
time (sec)	N/A	0.050	0.070	0.063	0.935	0.844	0.000	0.425	0.352

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	12	19	18	20	0	19	14
normalized size	1	1.00	0.67	1.06	1.00	1.11	0.00	1.06	0.78
time (sec)	N/A	0.036	0.017	0.048	0.783	0.571	0.000	0.390	0.404

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	42	33	31	37	0	34	21
normalized size	1	1.00	1.27	1.00	0.94	1.12	0.00	1.03	0.64
time (sec)	N/A	0.055	0.040	0.057	0.300	0.597	0.000	0.474	0.336

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	25	29	42	24	0	37	35
normalized size	1	1.00	0.83	0.97	1.40	0.80	0.00	1.23	1.17
time (sec)	N/A	0.080	0.051	0.046	0.295	0.786	0.000	0.472	0.382

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	60	55	56	83	0	50	40
normalized size	1	1.00	1.30	1.20	1.22	1.80	0.00	1.09	0.87
time (sec)	N/A	0.096	0.156	0.055	0.469	1.269	0.000	0.487	0.408

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	41	45	70	53	0	59	45
normalized size	1	1.00	1.02	1.12	1.75	1.32	0.00	1.48	1.12
time (sec)	N/A	0.081	0.082	0.055	1.718	2.405	0.000	0.368	0.455

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	49	28	27	43	0	28	20
normalized size	1	1.00	1.48	0.85	0.82	1.30	0.00	0.85	0.61
time (sec)	N/A	0.036	0.072	0.052	0.574	1.708	0.000	0.566	0.366

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	190	338	0	332	0	226	1666
normalized size	1	1.00	1.68	2.99	0.00	2.94	0.00	2.00	14.74
time (sec)	N/A	0.418	1.134	0.058	0.000	1.214	0.000	0.517	1.097

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	65	56	66	0	66	115
normalized size	1	1.00	0.81	1.14	0.98	1.16	0.00	1.16	2.02
time (sec)	N/A	0.084	0.096	0.053	0.504	0.928	0.000	0.506	0.603

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	85	129	0	203	0	111	77
normalized size	1	1.00	1.39	2.11	0.00	3.33	0.00	1.82	1.26
time (sec)	N/A	0.224	0.177	0.050	0.000	0.948	0.000	0.484	0.548

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	22	0	22	48
normalized size	1	1.00	1.00	1.05	1.00	1.10	0.00	1.10	2.40
time (sec)	N/A	0.037	0.009	0.039	0.343	0.649	0.000	0.420	0.514

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	54	48	53	0	54	47
normalized size	1	1.00	0.93	1.00	0.89	0.98	0.00	1.00	0.87
time (sec)	N/A	0.058	0.068	0.048	0.657	0.926	0.000	0.434	0.503

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	67	78	0	230	0	91	86
normalized size	1	1.00	0.87	1.01	0.00	2.99	0.00	1.18	1.12
time (sec)	N/A	0.102	0.326	0.056	0.000	0.847	0.000	0.489	0.537

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	100	114	116	185	0	138	116
normalized size	1	1.00	1.08	1.23	1.25	1.99	0.00	1.48	1.25
time (sec)	N/A	0.183	0.565	0.064	0.873	0.595	0.000	0.460	0.584

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	112	153	0	456	0	210	183
normalized size	1	1.00	0.81	1.11	0.00	3.30	0.00	1.52	1.33
time (sec)	N/A	0.206	0.649	0.059	0.000	0.911	0.000	0.802	0.626

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	81	63	77	0	68	-1
normalized size	1	1.00	1.00	1.84	1.43	1.75	0.00	1.55	-0.02
time (sec)	N/A	0.049	0.072	0.493	2.536	0.864	0.000	0.692	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	30	46	109	0	34	-1
normalized size	1	1.00	1.00	0.81	1.24	2.95	0.00	0.92	-0.03
time (sec)	N/A	0.057	0.020	0.035	1.673	1.338	0.000	0.379	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	35	98	0	22	-1
normalized size	1	1.00	1.00	0.79	1.46	4.08	0.00	0.92	-0.04
time (sec)	N/A	0.058	0.013	0.039	1.177	0.705	0.000	0.404	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	363	546	0	0	0	0	-1
normalized size	1	1.00	1.78	2.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.583	2.781	0.398	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.105	2.469	1.331	0.000	3.215	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [17] had the largest ratio of [.6154]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.00	13	0.385
2	A	5	4	1.00	13	0.308
3	A	4	4	1.00	13	0.308
4	A	4	4	1.00	11	0.364
5	A	5	5	1.00	11	0.454
6	A	5	4	1.00	13	0.308
7	A	6	5	1.00	13	0.385
8	A	6	5	1.00	13	0.385
9	A	3	2	1.00	13	0.154
10	A	6	6	1.00	13	0.462
11	A	3	2	1.00	13	0.154
12	A	6	6	1.00	13	0.462
13	A	4	4	1.00	11	0.364
14	A	3	2	1.00	11	0.182
15	A	7	6	1.00	13	0.462
16	A	4	3	1.00	13	0.231
17	A	12	8	1.00	13	0.615
18	A	5	4	1.00	13	0.308
19	A	4	4	1.00	13	0.308
20	A	3	3	1.00	13	0.231
21	A	9	7	1.00	25	0.280
22	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int \frac{\tan^4(x)}{a+a\cos(x)}\,dx$

Optimal. Leaf size=33

$$\frac{\tan^3(x)}{3a}+\frac{\tanh^{-1}(\sin(x))}{2a}-\frac{\tan(x)\sec(x)}{2a}$$

```
[Out] 1/2*arctanh(sin(x))/a-1/2*sec(x)*tan(x)/a+1/3*tan(x)^3/a
```

Rubi [A] time = 0.08, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2706, 2607, 30, 2611, 3770}

$$\frac{\tan^3(x)}{3a}+\frac{\tanh^{-1}(\sin(x))}{2a}-\frac{\tan(x)\sec(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^4/(a + a*Cos[x]),x]
```

```
[Out] ArcTanh[Sin[x]]/(2*a) - (Sec[x]*Tan[x])/(2*a) + Tan[x]^3/(3*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^4(x)}{a+a\cos(x)}\,dx &= -\frac{\int \sec(x)\tan^2(x)\,dx}{a}+\frac{\int \sec^2(x)\tan^2(x)\,dx}{a}\\ &= -\frac{\sec(x)\tan(x)}{2a}+\frac{\int \sec(x)\,dx}{2a}+\frac{\text{Subst}\left(\int x^2\,dx,x,\tan(x)\right)}{a}\\ &= \frac{\tanh^{-1}(\sin(x))}{2a}-\frac{\sec(x)\tan(x)}{2a}+\frac{\tan^3(x)}{3a}\end{aligned}$$

Mathematica [B] time = 0.13, size = 105, normalized size = 3.18

$$-\frac{\sec^3(x)\left(2(-3\sin(x)+3\sin(2x)+\sin(3x))+9\cos(x)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)+3\,\mathrm{co}}{24a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^4/(a + a*Cos[x]),x]
```

```
[Out] -1/24*(Sec[x]^3*(9*Cos[x]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/
2]]) + 3*Cos[3*x]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) + 2
*(-3*Sin[x] + 3*Sin[2*x] + Sin[3*x])))/a
```

fricas [A] time = 0.79, size = 50, normalized size = 1.52

$$\frac{3\,\cos(x)^3\log\left(\sin(x)+1\right)-3\,\cos(x)^3\log\left(-\sin(x)+1\right)-2\left(2\,\cos(x)^2+3\,\cos(x)-2\right)\sin(x)}{12\,a\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4/(a+a*cos(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*cos(x)^3*log(sin(x) + 1) - 3*cos(x)^3*log(-sin(x) + 1) - 2*(2*cos(x
)^2 + 3*cos(x) - 2)*sin(x))/(a*cos(x)^3)
```

giac [B] time = 0.71, size = 65, normalized size = 1.97

$$\frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)}{2\,a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)}{2\,a}-\frac{3\,\tan\left(\frac{1}{2}\,x\right)^5+8\,\tan\left(\frac{1}{2}\,x\right)^3-3\,\tan\left(\frac{1}{2}\,x\right)}{3\left(\tan\left(\frac{1}{2}\,x\right)^2-1\right)^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4/(a+a*cos(x)),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(tan(1/2*x) + 1))/a - 1/2*log(abs(tan(1/2*x) - 1))/a - 1/3*(3*ta
n(1/2*x)^5 + 8*tan(1/2*x)^3 - 3*tan(1/2*x))/((tan(1/2*x)^2 - 1)^3*a)
```

maple [B] time = 0.07, size = 103, normalized size = 3.12

$$-\frac{1}{3a\left(\tan\left(\frac{x}{2}\right)-1\right)^3}-\frac{1}{a\left(\tan\left(\frac{x}{2}\right)-1\right)^2}-\frac{1}{2a\left(\tan\left(\frac{x}{2}\right)-1\right)}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{2a}-\frac{1}{3a\left(\tan\left(\frac{x}{2}\right)+1\right)^3}+\frac{1}{a\left(\tan\left(\frac{x}{2}\right)+1\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^4/(a+a*cos(x)),x)
```

```
[Out] -1/3/a/(tan(1/2*x)-1)^3-1/a/(tan(1/2*x)-1)^2-1/2/a/(tan(1/2*x)-1)-1/2/a*ln(
tan(1/2*x)-1)-1/3/a/(tan(1/2*x)+1)^3+1/a/(tan(1/2*x)+1)^2-1/2/a/(tan(1/2*x)
+1)+1/2/a*ln(tan(1/2*x)+1)
```

maxima [B] time = 0.30, size = 115, normalized size = 3.48

$$-\frac{\frac{3\,\sin(x)}{\cos(x)+1}-\frac{8\,\sin(x)^3}{(\cos(x)+1)^3}-\frac{3\,\sin(x)^5}{(\cos(x)+1)^5}}{3\left(a-\frac{3\,a\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\,a\sin(x)^4}{(\cos(x)+1)^4}-\frac{a\sin(x)^6}{(\cos(x)+1)^6}\right)}+\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)}{2\,a}-\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4/(a+a*cos(x)),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sin(x)/(cos(x) + 1) - 8*sin(x)^3/(cos(x) + 1)^3 - 3*sin(x)^5/(cos(x
) + 1)^5)/(a - 3*a*sin(x)^2/(cos(x) + 1)^2 + 3*a*sin(x)^4/(cos(x) + 1)^4 -
a*sin(x)^6/(cos(x) + 1)^6) + 1/2*log(sin(x)/(cos(x) + 1) + 1)/a - 1/2*log(s
in(x)/(cos(x) + 1) - 1)/a
```

mupad [B] time = 0.46, size = 46, normalized size = 1.39

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{a}-\frac{\tan\left(\frac{x}{2}\right)^5+\frac{8\tan\left(\frac{x}{2}\right)^3}{3}-\tan\left(\frac{x}{2}\right)}{a\left(\tan\left(\frac{x}{2}\right)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^4/(a + a*cos(x)),x)
```

```
[Out] atanh(tan(x/2))/a - ((8*tan(x/2)^3)/3 - tan(x/2) + tan(x/2)^5)/(a*(tan(x/2)
^2 - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^4(x)}{\cos(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**4/(a+a*cos(x)),x)
```

```
[Out] Integral(tan(x)**4/(cos(x) + 1), x)/a
```

3.2 $\int \frac{\tan^3(x)}{a+a\cos(x)}\,dx$

Optimal. Leaf size=19

$$\frac{\sec^2(x)}{2a}-\frac{\sec(x)}{a}$$

```
[Out] -sec(x)/a+1/2*sec(x)^2/a
```

Rubi [A] time = 0.06, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2706, 2606, 30, 8}

$$\frac{\sec^2(x)}{2a}-\frac{\sec(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3/(a + a*Cos[x]),x]
```

```
[Out] -(Sec[x]/a) + Sec[x]^2/(2*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^3(x)}{a+a\cos(x)}\,dx = -\frac{\int \sec(x)\tan(x)\,dx}{a} + \frac{\int \sec^2(x)\tan(x)\,dx}{a}$$
$$= -\frac{\text{Subst}(\int 1\,dx, x, \sec(x))}{a} + \frac{\text{Subst}(\int x\,dx, x, \sec(x))}{a}$$
$$= -\frac{\sec(x)}{a} + \frac{\sec^2(x)}{2a}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 0.89

$$\frac{2\sin^4\left(\frac{x}{2}\right)\sec^2(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3/(a + a*Cos[x]),x]
[Out] (2*Sec[x]^2*Sin[x/2]^4)/a
```

fricas [A] time = 0.74, size = 15, normalized size = 0.79

$$-\frac{2\cos(x)-1}{2a\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+a*cos(x)),x, algorithm="fricas")
[Out] -1/2*(2*cos(x) - 1)/(a*cos(x)^2)
```

giac [A] time = 0.36, size = 15, normalized size = 0.79

$$-\frac{2\cos(x)-1}{2a\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+a*cos(x)),x, algorithm="giac")
[Out] -1/2*(2*cos(x) - 1)/(a*cos(x)^2)
```

maple [A] time = 0.05, size = 18, normalized size = 0.95

$$\frac{\frac{1}{2\cos(x)^2}-\frac{1}{\cos(x)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a+a*cos(x)),x)
```

```
[Out] 1/a*(1/2/cos(x)^2-1/cos(x))
```

maxima [A] time = 0.66, size = 15, normalized size = 0.79

$$-\frac{2\cos(x)-1}{2\,a\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+a*cos(x)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*cos(x) - 1)/(a*cos(x)^2)
```

mupad [B] time = 0.34, size = 13, normalized size = 0.68

$$-\frac{\cos(x)-\frac{1}{2}}{a\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a + a*cos(x)),x)
```

```
[Out] -(cos(x) - 1/2)/(a*cos(x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^3(x)}{\cos(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3/(a+a*cos(x)),x)
```

```
[Out] Integral(tan(x)**3/(cos(x) + 1), x)/a
```

3.3 $\int \frac{\tan^2(x)}{a+a\cos(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\tan(x)}{a}-\frac{\tanh^{-1}(\sin(x))}{a}$$

```
[Out] -arctanh(sin(x))/a+tan(x)/a
```

Rubi [A] time = 0.05, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2706, 3767, 8, 3770}

$$\frac{\tan(x)}{a}-\frac{\tanh^{-1}(\sin(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2/(a + a*Cos[x]),x]
```

```
[Out] -(ArcTanh[Sin[x]]/a) + Tan[x]/a
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^2(x)}{a+a\cos(x)}\,dx = -\frac{\int \sec(x)\,dx}{a} + \frac{\int \sec^2(x)\,dx}{a}$$
$$= -\frac{\tanh^{-1}(\sin(x))}{a} - \frac{\text{Subst}(\int 1\,dx, x, -\tan(x))}{a}$$
$$= -\frac{\tanh^{-1}(\sin(x))}{a} + \frac{\tan(x)}{a}$$

Mathematica [B] time = 0.07, size = 39, normalized size = 2.60

$$\frac{\tan(x) + \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2/(a + a*Cos[x]),x]
[Out] (Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]] + Tan[x])/a
```

fricas [B] time = 0.84, size = 33, normalized size = 2.20

$$-\frac{\cos(x)\log(\sin(x)+1) - \cos(x)\log(-\sin(x)+1) - 2\sin(x)}{2\,a\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*cos(x)),x, algorithm="fricas")
[Out] -1/2*(cos(x)*log(sin(x) + 1) - cos(x)*log(-sin(x) + 1) - 2*sin(x))/(a*cos(x
))
```

giac [B] time = 0.43, size = 45, normalized size = 3.00

$$-\frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)}{a} + \frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)}{a} - \frac{2\tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right)^2-1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*cos(x)),x, algorithm="giac")
[Out] -log(abs(tan(1/2*x) + 1))/a + log(abs(tan(1/2*x) - 1))/a - 2*tan(1/2*x)/((t
an(1/2*x)^2 - 1)*a)
```

maple [B] time = 0.06, size = 51, normalized size = 3.40

$$-\frac{1}{a\left(\tan\left(\frac{x}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{a}-\frac{1}{a\left(\tan\left(\frac{x}{2}\right)+1\right)}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a+a*cos(x)),x)
```

```
[Out] -1/a/(tan(1/2*x)-1)+1/a*ln(tan(1/2*x)-1)-1/a/(tan(1/2*x)+1)-1/a*ln(tan(1/2*
x)+1)
```

maxima [B] time = 0.94, size = 61, normalized size = 4.07

$$-\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)}{a}+\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)}{a}+\frac{2\sin(x)}{\left(a-\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+a*cos(x)),x, algorithm="maxima")
```

```
[Out] -log(sin(x)/(cos(x) + 1) + 1)/a + log(sin(x)/(cos(x) + 1) - 1)/a + 2*sin(x)
/((a - a*sin(x)^2/(cos(x) + 1)^2)*(cos(x) + 1))
```

mupad [B] time = 0.35, size = 30, normalized size = 2.00

$$-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{a}-\frac{2\tan\left(\frac{x}{2}\right)}{a\left(\tan\left(\frac{x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a + a*cos(x)),x)
```

```
[Out] - (2*atanh(tan(x/2)))/a - (2*tan(x/2))/(a*(tan(x/2)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan^2(x)}{\cos(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2/(a+a*cos(x)),x)
```

```
[Out] Integral(tan(x)**2/(cos(x) + 1), x)/a
```

3.4 $\int \frac{\tan(x)}{a+a\cos(x)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(\cos(x)+1)}{a}-\frac{\log(\cos(x))}{a}$$

```
[Out] -ln(cos(x))/a+ln(1+cos(x))/a
```

Rubi [A] time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2707, 36, 29, 31}

$$\frac{\log(\cos(x)+1)}{a}-\frac{\log(\cos(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(a + a*Cos[x]),x]
```

```
[Out] -(Log[Cos[x]]/a) + Log[1 + Cos[x]]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan(x)}{a+a\cos(x)}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, a\cos(x)\right)$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, a\cos(x)\right)}{a} + \frac{\operatorname{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\cos(x)\right)}{a}$$
$$= -\frac{\log(\cos(x))}{a} + \frac{\log(1+\cos(x))}{a}$$

Mathematica [A] time = 0.02, size = 12, normalized size = 0.67

$$\frac{2\tanh^{-1}(2\cos(x)+1)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(a + a*Cos[x]),x]
[Out] (2*ArcTanh[1 + 2*Cos[x]])/a
```

fricas [A] time = 0.57, size = 20, normalized size = 1.11

$$-\frac{\log\left(-\cos(x)\right) - \log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*cos(x)),x, algorithm="fricas")
[Out] -(log(-cos(x)) - log(1/2*cos(x) + 1/2))/a
```

giac [A] time = 0.39, size = 19, normalized size = 1.06

$$\frac{\log\left(\cos(x)+1\right)}{a} - \frac{\log\left(|\cos(x)|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*cos(x)),x, algorithm="giac")
[Out] log(cos(x) + 1)/a - log(abs(cos(x)))/a
```

maple [A] time = 0.05, size = 19, normalized size = 1.06

$$-\frac{\ln\left(\cos(x)\right)}{a} + \frac{\ln\left(\cos(x)+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+a*cos(x)),x)
```

```
[Out] -ln(cos(x))/a+ln(cos(x)+1)/a
```

maxima [A] time = 0.78, size = 18, normalized size = 1.00

$$\frac{\log\left(\cos(x)+1\right)}{a}-\frac{\log\left(\cos(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*cos(x)),x, algorithm="maxima")
```

```
[Out] log(cos(x) + 1)/a - log(cos(x))/a
```

mupad [B] time = 0.40, size = 14, normalized size = 0.78

$$-\frac{\ln\left(\tan\left(\frac{x}{2}\right)^2-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + a*cos(x)),x)
```

```
[Out] -log(tan(x/2)^2 - 1)/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\tan(x)}{\cos(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+a*cos(x)),x)
```

```
[Out] Integral(tan(x)/(cos(x) + 1), x)/a
```

3.5 $\int \frac{\cot(x)}{a+a\cos(x)}\,dx$

Optimal. Leaf size=33

$$-\frac{\csc^2(x)}{2a}-\frac{\tanh^{-1}(\cos(x))}{2a}+\frac{\cot(x)\csc(x)}{2a}$$

```
[Out] -1/2*arctanh(cos(x))/a+1/2*cot(x)*csc(x)/a-1/2*csc(x)^2/a
```

Rubi [A] time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {2706, 2606, 30, 2611, 3770}

$$-\frac{\csc^2(x)}{2a}-\frac{\tanh^{-1}(\cos(x))}{2a}+\frac{\cot(x)\csc(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a + a*Cos[x]),x]
```

```
[Out] -ArcTanh[Cos[x]]/(2*a) + (Cot[x]*Csc[x])/(2*a) - Csc[x]^2/(2*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
```

```
- Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)}{a+a\cos(x)}\,dx &= -\frac{\int \cot^2(x)\csc(x)\,dx}{a}+\frac{\int \cot(x)\csc^2(x)\,dx}{a}\\ &= \frac{\cot(x)\csc(x)}{2a}+\frac{\int \csc(x)\,dx}{2a}-\frac{\text{Subst}(\int x\,dx, x, \csc(x))}{a}\\ &= -\frac{\tanh^{-1}(\cos(x))}{2a}+\frac{\cot(x)\csc(x)}{2a}-\frac{\csc^2(x)}{2a}\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 1.27

$$-\frac{2\cos^2\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)+1}{2a(\cos(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a + a*Cos[x]),x]
[Out] -1/2*(1 + 2*Cos[x/2]^2*(Log[Cos[x/2]] - Log[Sin[x/2]]))/(a*(1 + Cos[x]))
```

fricas [A] time = 0.60, size = 37, normalized size = 1.12

$$-\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)+2}{4\,(a\cos(x)+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*cos(x)),x, algorithm="fricas")
[Out] -1/4*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*cos(x) + 1
/2) + 2)/(a*cos(x) + a)
```

giac [A] time = 0.47, size = 34, normalized size = 1.03

$$-\frac{\log\left(\cos(x)+1\right)}{4\,a}+\frac{\log\left(-\cos(x)+1\right)}{4\,a}-\frac{1}{2\,a(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*cos(x)),x, algorithm="giac")
```

```
[Out] -1/4*log(cos(x) + 1)/a + 1/4*log(-cos(x) + 1)/a - 1/2/(a*(cos(x) + 1))
```

maple [A] time = 0.06, size = 33, normalized size = 1.00

$$\frac{\ln\left(-1+\cos(x)\right)}{4a}-\frac{1}{2a\left(\cos(x)+1\right)}-\frac{\ln\left(\cos(x)+1\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+a*cos(x)),x)
```

```
[Out] 1/4/a*ln(-1+cos(x))-1/2/a/(cos(x)+1)-1/4*ln(cos(x)+1)/a
```

maxima [A] time = 0.30, size = 31, normalized size = 0.94

$$-\frac{\log\left(\cos(x)+1\right)}{4\,a}+\frac{\log\left(\cos(x)-1\right)}{4\,a}-\frac{1}{2\left(a\cos(x)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*cos(x)),x, algorithm="maxima")
```

```
[Out] -1/4*log(cos(x) + 1)/a + 1/4*log(cos(x) - 1)/a - 1/2/(a*cos(x) + a)
```

mupad [B] time = 0.34, size = 21, normalized size = 0.64

$$\frac{2\,\ln\left(\tan\left(\frac{x}{2}\right)\right)-\tan\left(\frac{x}{2}\right)^2}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + a*cos(x)),x)
```

```
[Out] (2*log(tan(x/2)) - tan(x/2)^2)/(4*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cot(x)}{\cos(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+a*cos(x)),x)
```

```
[Out] Integral(cot(x)/(cos(x) + 1), x)/a
```

3.6 $\int \frac{\cot^2(x)}{a+a\cos(x)}\,dx$

Optimal. Leaf size=30

$$-\frac{\cot^3(x)}{3a}+\frac{\csc^3(x)}{3a}-\frac{\csc(x)}{a}$$

```
[Out] -1/3*cot(x)^3/a-csc(x)/a+1/3*csc(x)^3/a
```

Rubi [A] time = 0.08, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2706, 2607, 30, 2606}

$$-\frac{\cot^3(x)}{3a}+\frac{\csc^3(x)}{3a}-\frac{\csc(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2/(a + a*Cos[x]),x]
```

```
[Out] -Cot[x]^3/(3*a) - Csc[x]/a + Csc[x]^3/(3*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
```

```
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^2(x)}{a+a\cos(x)}\,dx &= -\frac{\int \cot^3(x)\csc(x)\,dx}{a}+\frac{\int \cot^2(x)\csc^2(x)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int x^2\,dx, x, -\cot(x)\right)}{a}+\frac{\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \csc(x)\right)}{a}\\ &= -\frac{\cot^3(x)}{3a}-\frac{\csc(x)}{a}+\frac{\csc^3(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.05, size = 25, normalized size = 0.83

$$\frac{(-4\cos(x)+\cos(2x)-3)\csc(x)}{6a(\cos(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2/(a + a*Cos[x]),x]
[Out] ((-3 - 4*Cos[x] + Cos[2*x])*Csc[x])/(6*a*(1 + Cos[x]))
```

fricas [A] time = 0.79, size = 24, normalized size = 0.80

$$\frac{\cos(x)^2-2\cos(x)-2}{3\,(a\cos(x)+a)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*cos(x)),x, algorithm="fricas")
[Out] 1/3*(cos(x)^2 - 2*cos(x) - 2)/((a*cos(x) + a)*sin(x))
```

giac [A] time = 0.47, size = 37, normalized size = 1.23

$$\frac{a^2\tan\left(\frac{1}{2}x\right)^3-6\,a^2\tan\left(\frac{1}{2}x\right)}{12\,a^3}-\frac{1}{4\,a\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*cos(x)),x, algorithm="giac")
```

```
[Out] 1/12*(a^2*tan(1/2*x)^3 - 6*a^2*tan(1/2*x))/a^3 - 1/4/(a*tan(1/2*x))
```

maple [A] time = 0.05, size = 29, normalized size = 0.97

$$\frac{\frac{\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{3}-2\tan\left(\frac{x}{2}\right)-\frac{1}{\tan\left(\frac{x}{2}\right)}}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a+a*cos(x)),x)
```

```
[Out] 1/4/a*(1/3*tan(1/2*x)^3-2*tan(1/2*x)-1/tan(1/2*x))
```

maxima [A] time = 0.30, size = 42, normalized size = 1.40

$$-\frac{\frac{6\sin(x)}{\cos(x)+1}-\frac{\sin(x)^{3}}{(\cos(x)+1)^{3}}}{12a}-\frac{\cos(x)+1}{4a\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+a*cos(x)),x, algorithm="maxima")
```

```
[Out] -1/12*(6*sin(x)/(cos(x) + 1) - sin(x)^3/(cos(x) + 1)^3)/a - 1/4*(cos(x) + 1
)/(a*sin(x))
```

mupad [B] time = 0.38, size = 35, normalized size = 1.17

$$\frac{4\cos\left(\frac{x}{2}\right)^{4}-8\cos\left(\frac{x}{2}\right)^{2}+1}{12a\cos\left(\frac{x}{2}\right)^{3}\sin\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a + a*cos(x)),x)
```

```
[Out] (4*cos(x/2)^4 - 8*cos(x/2)^2 + 1)/(12*a*cos(x/2)^3*sin(x/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cot^{2}(x)}{\cos(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2/(a+a*cos(x)),x)
```

```
[Out] Integral(cot(x)**2/(cos(x) + 1), x)/a
```

3.7 $\int \frac{\cot^3(x)}{a+a\cos(x)}\,dx$

Optimal. Leaf size=46

$$-\frac{\cot^4(x)}{4a}+\frac{3\tanh^{-1}(\cos(x))}{8a}+\frac{\cot^3(x)\csc(x)}{4a}-\frac{3\cot(x)\csc(x)}{8a}$$

```
[Out] 3/8*arctanh(cos(x))/a-1/4*cot(x)^4/a-3/8*cot(x)*csc(x)/a+1/4*cot(x)^3*csc(x
)/a
```

Rubi [A] time = 0.10, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2706, 2607, 30, 2611, 3770}

$$-\frac{\cot^4(x)}{4a}+\frac{3\tanh^{-1}(\cos(x))}{8a}+\frac{\cot^3(x)\csc(x)}{4a}-\frac{3\cot(x)\csc(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3/(a + a*Cos[x]),x]
```

```
[Out] (3*ArcTanh[Cos[x]])/(8*a) - Cot[x]^4/(4*a) - (3*Cot[x]*Csc[x])/(8*a) + (Cot
[x]^3*Csc[x])/(4*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^3(x)}{a+a\cos(x)}\,dx &= -\frac{\int \cot^4(x)\csc(x)\,dx}{a}+\frac{\int \cot^3(x)\csc^2(x)\,dx}{a}\\
&= \frac{\cot^3(x)\csc(x)}{4a}+\frac{3\int \cot^2(x)\csc(x)\,dx}{4a}-\frac{\mathrm{Subst}\left(\int x^3\,dx,x,-\cot(x)\right)}{a}\\
&= -\frac{\cot^4(x)}{4a}-\frac{3\cot(x)\csc(x)}{8a}+\frac{\cot^3(x)\csc(x)}{4a}-\frac{3\int \csc(x)\,dx}{8a}\\
&= \frac{3\tanh^{-1}(\cos(x))}{8a}-\frac{\cot^4(x)}{4a}-\frac{3\cot(x)\csc(x)}{8a}+\frac{\cot^3(x)\csc(x)}{4a}
\end{aligned}
$$

Mathematica [A] time = 0.16, size = 60, normalized size = 1.30

$$
-\frac{2\cot^2\left(\frac{x}{2}\right)+\sec^2\left(\frac{x}{2}\right)-12\cos^2\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)-8}{16a(\cos(x)+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3/(a + a*Cos[x]),x]
[Out] -1/16*(-8 + 2*Cot[x/2]^2 - 12*Cos[x/2]^2*(Log[Cos[x/2]] - Log[Sin[x/2]]) +
Sec[x/2]^2)/(a*(1 + Cos[x]))
```

fricas [B] time = 1.27, size = 83, normalized size = 1.80

$$
\frac{10\,\cos(x)^2+3\left(\cos(x)^3+\cos(x)^2-\cos(x)-1\right)\log\left(\frac{1}{2}\,\cos(x)+\frac{1}{2}\right)-3\left(\cos(x)^3+\cos(x)^2-\cos(x)-1\right)\log\left(-\frac{1}{2}\right.}{16\left(a\cos(x)^3+a\cos(x)^2-a\cos(x)-a\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+a*cos(x)),x, algorithm="fricas")
```

```
[Out] 1/16*(10*cos(x)^2 + 3*(cos(x)^3 + cos(x)^2 - cos(x) - 1)*log(1/2*cos(x) + 1
/2) - 3*(cos(x)^3 + cos(x)^2 - cos(x) - 1)*log(-1/2*cos(x) + 1/2) + 2*cos(x
) - 4)/(a*cos(x)^3 + a*cos(x)^2 - a*cos(x) - a)
```

giac [A] time = 0.49, size = 50, normalized size = 1.09

$$\frac{3\,\log\left(\cos(x)+1\right)}{16\,a}-\frac{3\,\log\left(-\cos(x)+1\right)}{16\,a}+\frac{5\,\cos(x)^{2}+\cos(x)-2}{8\,a(\cos(x)+1)^{2}(\cos(x)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+a*cos(x)),x, algorithm="giac")
```

```
[Out] 3/16*log(cos(x) + 1)/a - 3/16*log(-cos(x) + 1)/a + 1/8*(5*cos(x)^2 + cos(x)
 - 2)/(a*(cos(x) + 1)^2*(cos(x) - 1))
```

maple [A] time = 0.06, size = 55, normalized size = 1.20

$$\frac{1}{8a\left(-1+\cos(x)\right)}-\frac{3\ln\left(-1+\cos(x)\right)}{16a}-\frac{1}{8a\left(\cos(x)+1\right)^{2}}+\frac{1}{2a\left(\cos(x)+1\right)}+\frac{3\ln\left(\cos(x)+1\right)}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a+a*cos(x)),x)
```

```
[Out] 1/8/a/(-1+cos(x))-3/16/a*ln(-1+cos(x))-1/8/a/(cos(x)+1)^2+1/2/a/(cos(x)+1)+
3/16*ln(cos(x)+1)/a
```

maxima [A] time = 0.47, size = 56, normalized size = 1.22

$$\frac{5\,\cos(x)^{2}+\cos(x)-2}{8\left(a\cos(x)^{3}+a\cos(x)^{2}-a\cos(x)-a\right)}+\frac{3\,\log\left(\cos(x)+1\right)}{16\,a}-\frac{3\,\log\left(\cos(x)-1\right)}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+a*cos(x)),x, algorithm="maxima")
```

```
[Out] 1/8*(5*cos(x)^2 + cos(x) - 2)/(a*cos(x)^3 + a*cos(x)^2 - a*cos(x) - a) + 3/
16*log(cos(x) + 1)/a - 3/16*log(cos(x) - 1)/a
```

mupad [B] time = 0.41, size = 40, normalized size = 0.87

$$-\frac{\tan\left(\frac{x}{2}\right)^{6}-6\tan\left(\frac{x}{2}\right)^{4}+12\tan\left(\frac{x}{2}\right)^{2}\,\ln\left(\tan\left(\frac{x}{2}\right)\right)+2}{32\,a\tan\left(\frac{x}{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a + a*cos(x)),x)
```

```
[Out] -(tan(x/2)^6 - 6*tan(x/2)^4 + 12*tan(x/2)^2*log(tan(x/2)) + 2)/(32*a*tan(x/
2)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^3(x)}{\cos(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3/(a+a*cos(x)),x)
```

```
[Out] Integral(cot(x)**3/(cos(x) + 1), x)/a
```

3.8 $\int \frac{\cot^4(x)}{a+a\cos(x)}\,dx$

Optimal. Leaf size=40

$$-\frac{\cot^5(x)}{5a}+\frac{\csc^5(x)}{5a}-\frac{2\csc^3(x)}{3a}+\frac{\csc(x)}{a}$$

```
[Out] -1/5*cot(x)^5/a+csc(x)/a-2/3*csc(x)^3/a+1/5*csc(x)^5/a
```

Rubi [A] time = 0.08, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2706, 2607, 30, 2606, 194}

$$-\frac{\cot^5(x)}{5a}+\frac{\csc^5(x)}{5a}-\frac{2\csc^3(x)}{3a}+\frac{\csc(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^4/(a + a*Cos[x]),x]
```

```
[Out] -Cot[x]^5/(5*a) + Csc[x]/a - (2*Csc[x]^3)/(3*a) + Csc[x]^5/(5*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^4(x)}{a+a\cos(x)}\,dx &= -\frac{\int \cot^5(x)\csc(x)\,dx}{a}+\frac{\int \cot^4(x)\csc^2(x)\,dx}{a}\\
&= \frac{\operatorname{Subst}\left(\int x^4\,dx, x, -\cot(x)\right)}{a}+\frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \csc(x)\right)}{a}\\
&= -\frac{\cot^5(x)}{5a}+\frac{\operatorname{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \csc(x)\right)}{a}\\
&= -\frac{\cot^5(x)}{5a}+\frac{\csc(x)}{a}-\frac{2\csc^3(x)}{3a}+\frac{\csc^5(x)}{5a}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 41, normalized size = 1.02

$$-\frac{(8\cos(x)+36\cos(2x)+24\cos(3x)-3\cos(4x)-25)\csc^3(x)}{120a(\cos(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^4/(a + a*Cos[x]),x]
```

```
[Out] -1/120*((-25 + 8*Cos[x] + 36*Cos[2*x] + 24*Cos[3*x] - 3*Cos[4*x])*Csc[x]^3)
/(a*(1 + Cos[x]))
```

fricas [A] time = 2.40, size = 53, normalized size = 1.32

$$-\frac{3\cos(x)^4-12\cos(x)^3-12\cos(x)^2+8\cos(x)+8}{15\left(a\cos(x)^3+a\cos(x)^2-a\cos(x)-a\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4/(a+a*cos(x)),x, algorithm="fricas")
```

```
[Out] -1/15*(3*cos(x)^4 - 12*cos(x)^3 - 12*cos(x)^2 + 8*cos(x) + 8)/((a*cos(x)^3
+ a*cos(x)^2 - a*cos(x) - a)*sin(x))
```

giac [A] time = 0.37, size = 59, normalized size = 1.48

$$\frac{12\,\tan\left(\frac{1}{2}\,x\right)^{2}-1}{48\,a\tan\left(\frac{1}{2}\,x\right)^{3}}+\frac{3\,a^{4}\tan\left(\frac{1}{2}\,x\right)^{5}-20\,a^{4}\tan\left(\frac{1}{2}\,x\right)^{3}+90\,a^{4}\tan\left(\frac{1}{2}\,x\right)}{240\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4/(a+a*cos(x)),x, algorithm="giac")
```

```
[Out] 1/48*(12*tan(1/2*x)^2 - 1)/(a*tan(1/2*x)^3) + 1/240*(3*a^4*tan(1/2*x)^5 - 2
0*a^4*tan(1/2*x)^3 + 90*a^4*tan(1/2*x))/a^5
```

maple [A] time = 0.06, size = 45, normalized size = 1.12

$$\frac{\frac{\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{5}-\frac{4\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{3}+6\tan\left(\frac{x}{2}\right)-\frac{1}{3\tan\left(\frac{x}{2}\right)^{3}}+\frac{4}{\tan\left(\frac{x}{2}\right)}}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4/(a+a*cos(x)),x)
```

```
[Out] 1/16/a*(1/5*tan(1/2*x)^5-4/3*tan(1/2*x)^3+6*tan(1/2*x)-1/3/tan(1/2*x)^3+4/t
an(1/2*x))
```

maxima [B] time = 1.72, size = 70, normalized size = 1.75

$$\frac{\frac{90\,\sin(x)}{\cos(x)+1}-\frac{20\,\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{3\,\sin(x)^{5}}{(\cos(x)+1)^{5}}}{240\,a}+\frac{\left(\frac{12\,\sin(x)^{2}}{(\cos(x)+1)^{2}}-1\right)(\cos(x)+1)^{3}}{48\,a\sin(x)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4/(a+a*cos(x)),x, algorithm="maxima")
```

```
[Out] 1/240*(90*sin(x)/(cos(x) + 1) - 20*sin(x)^3/(cos(x) + 1)^3 + 3*sin(x)^5/(co
s(x) + 1)^5)/a + 1/48*(12*sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)^3/(a*si
n(x)^3)
```

mupad [B] time = 0.45, size = 45, normalized size = 1.12

$$\frac{3\tan\left(\frac{x}{2}\right)^{8}-20\tan\left(\frac{x}{2}\right)^{6}+90\tan\left(\frac{x}{2}\right)^{4}+60\tan\left(\frac{x}{2}\right)^{2}-5}{240\,a\tan\left(\frac{x}{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4/(a + a*cos(x)),x)
```

```
[Out] (60*tan(x/2)^2 + 90*tan(x/2)^4 - 20*tan(x/2)^6 + 3*tan(x/2)^8 - 5)/(240*a*t
an(x/2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cot^4(x)}{\cos(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**4/(a+a*cos(x)),x)
```

```
[Out] Integral(cot(x)**4/(cos(x) + 1), x)/a
```

3.9 $\int \frac{\tan(3x)}{(1+\cos(3x))^2} dx$

Optimal. Leaf size=33

$$-\frac{1}{3(\cos(3x)+1)} - \frac{1}{3}\log(\cos(3x)) + \frac{1}{3}\log(\cos(3x)+1)$$

```
[Out] -1/3/(1+cos(3*x))-1/3*ln(cos(3*x))+1/3*ln(1+cos(3*x))
```

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2707, 44}

$$-\frac{1}{3(\cos(3x)+1)} - \frac{1}{3}\log(\cos(3x)) + \frac{1}{3}\log(\cos(3x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[3*x]/(1 + Cos[3*x])^2,x]
```

```
[Out] -1/(3*(1 + Cos[3*x])) - Log[Cos[3*x]]/3 + Log[1 + Cos[3*x]]/3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan(3x)}{(1+\cos(3x))^2}\,dx = -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x(1+x)^2}\,dx, x, \cos(3x)\right)\right)$$
$$= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{1}{x}-\frac{1}{(1+x)^2}\right)dx, x, \cos(3x)\right)\right)$$
$$= -\frac{1}{3(1+\cos(3x))}-\frac{1}{3}\log(\cos(3x))+\frac{1}{3}\log(1+\cos(3x))$$

Mathematica [A] time = 0.07, size = 49, normalized size = 1.48

$$\frac{\cos^4\left(\frac{3x}{2}\right)\left(8\log\left(\cos\left(\frac{3x}{2}\right)\right)-4\log(\cos(3x))\right)-2\cos^2\left(\frac{3x}{2}\right)}{3(\cos(3x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[3*x]/(1 + Cos[3*x])^2,x]
```

```
[Out] (-2*Cos[(3*x)/2]^2 + Cos[(3*x)/2]^4*(8*Log[Cos[(3*x)/2]] - 4*Log[Cos[3*x]])
)/(3*(1 + Cos[3*x])^2)
```

fricas [A] time = 1.71, size = 43, normalized size = 1.30

$$-\frac{(\cos(3x)+1)\log(-\cos(3x))-(\cos(3x)+1)\log\left(\frac{1}{2}\cos(3x)+\frac{1}{2}\right)+1}{3(\cos(3x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(3*x)/(1+cos(3*x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*((cos(3*x) + 1)*log(-cos(3*x)) - (cos(3*x) + 1)*log(1/2*cos(3*x) + 1/2
) + 1)/(cos(3*x) + 1)
```

giac [A] time = 0.57, size = 28, normalized size = 0.85

$$-\frac{1}{3(\cos(3x)+1)}+\frac{1}{3}\log(\cos(3x)+1)-\frac{1}{3}\log(|\cos(3x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(3*x)/(1+cos(3*x))^2,x, algorithm="giac")
```

```
[Out] -1/3/(cos(3*x) + 1) + 1/3*log(cos(3*x) + 1) - 1/3*log(abs(cos(3*x)))
```

maple [A] time = 0.05, size = 28, normalized size = 0.85

$$-\frac{1}{3\left(1+\cos\left(3x\right)\right)}-\frac{\ln\left(\cos\left(3x\right)\right)}{3}+\frac{\ln\left(1+\cos\left(3x\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(3*x)/(1+cos(3*x))^2,x)
```

```
[Out] -1/3/(1+cos(3*x))-1/3*ln(cos(3*x))+1/3*ln(1+cos(3*x))
```

maxima [A] time = 0.57, size = 27, normalized size = 0.82

$$-\frac{1}{3\left(\cos\left(3\,x\right)+1\right)}+\frac{1}{3}\,\log\left(\cos\left(3\,x\right)+1\right)-\frac{1}{3}\,\log\left(\cos\left(3\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(3*x)/(1+cos(3*x))^2,x, algorithm="maxima")
```

```
[Out] -1/3/(cos(3*x) + 1) + 1/3*log(cos(3*x) + 1) - 1/3*log(cos(3*x))
```

mupad [B] time = 0.37, size = 20, normalized size = 0.61

$$-\frac{\ln\left(\tan\left(\frac{3\,x}{2}\right)^2-1\right)}{3}-\frac{\tan\left(\frac{3\,x}{2}\right)^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(3*x)/(cos(3*x) + 1)^2,x)
```

```
[Out] - log(tan((3*x)/2)^2 - 1)/3 - tan((3*x)/2)^2/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan\left(3x\right)}{\left(\cos\left(3x\right)+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(3*x)/(1+cos(3*x))**2,x)
```

```
[Out] Integral(tan(3*x)/(cos(3*x) + 1)**2, x)
```

3.10 $\int \frac{\tan^4(x)}{a+b\cos(x)}\,dx$

Optimal. Leaf size=113

$$\frac{2(a-b)^{3/2}(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4}-\frac{b\tan(x)\sec(x)}{2a^2}+\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\sin(x))}{2a^4}-\frac{\left(4a^2-3b^2\right)\tan(x)}{3a^3}+\tan($$

```
[Out] 2*(a-b)^(3/2)*(a+b)^(3/2)*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/a^4+1/
2*b*(3*a^2-2*b^2)*arctanh(sin(x))/a^4-1/3*(4*a^2-3*b^2)*tan(x)/a^3-1/2*b*se
c(x)*tan(x)/a^2+1/3*sec(x)^2*tan(x)/a
```

Rubi [A] time = 0.42, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2725, 3055, 3001, 3770, 2659, 205}

$$-\frac{\left(4a^2-3b^2\right)\tan(x)}{3a^3}+\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\sin(x))}{2a^4}+\frac{2(a-b)^{3/2}(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4}-\frac{b\tan(x)\sec(x)}{2a^2}+\tan$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^4/(a + b*Cos[x]),x]
```

```
[Out] (2*(a - b)^(3/2)*(a + b)^(3/2)*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/
a^4 + (b*(3*a^2 - 2*b^2)*ArcTanh[Sin[x]])/(2*a^4) - ((4*a^2 - 3*b^2)*Tan[x]
)/(3*a^3) - (b*Sec[x]*Tan[x])/(2*a^2) + (Sec[x]^2*Tan[x])/(3*a)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2725

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m*Simp[8*a^2 -
```

```
 b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*m*(m - 2))*Sin[e +
 f*x]^2, x])/Sin[e + f*x]^2, x], x] - Simp[(b*(m - 2)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(m + 1))/(6*a^2*f*Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f, m
}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^4(x)}{a+b\cos(x)}\,dx = -\frac{b\sec(x)\tan(x)}{2a^2}+\frac{\sec^2(x)\tan(x)}{3a}-\frac{\int \frac{\left(2\left(4a^2-3b^2\right)-ab\cos(x)-3\left(2a^2-b^2\right)\cos^2(x)\right)\sec^2(x)}{a+b\cos(x)}dx}{6a^2}$$

$$= -\frac{\left(4a^2-3b^2\right)\tan(x)}{3a^3}-\frac{b\sec(x)\tan(x)}{2a^2}+\frac{\sec^2(x)\tan(x)}{3a}-\frac{\int \frac{\left(-3b\left(3a^2-2b^2\right)-3a\left(2a^2-b^2\right)\cos(x)\right)\sec(x}{a+b\cos(x)}}{6a^3}$$

$$= -\frac{\left(4a^2-3b^2\right)\tan(x)}{3a^3}-\frac{b\sec(x)\tan(x)}{2a^2}+\frac{\sec^2(x)\tan(x)}{3a}+\frac{\left(b\left(3a^2-2b^2\right)\right)\int \sec(x)\,dx}{2a^4}+\frac{(a^2}{}$$

$$= \frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\sin(x))}{2a^4}-\frac{\left(4a^2-3b^2\right)\tan(x)}{3a^3}-\frac{b\sec(x)\tan(x)}{2a^2}+\frac{\sec^2(x)\tan(x)}{3a}+\frac{(2}{}$$

$$= \frac{2(a-b)^{3/2}(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4}+\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\sin(x))}{2a^4}-\frac{\left(4a^2-3b^2\right)\tan(x}{3a^3}$$

Mathematica [A] time = 1.13, size = 190, normalized size = 1.68

$$-\frac{48\left(b^2-a^2\right)^{3/2}\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)+\sec^3(x)\left(2a\left(\left(4a^2-3b^2\right)\sin(3x)+3ab\sin(2x)-3b^2\sin(x)\right)+9b\left(3a^2-2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^4/(a + b*Cos[x]),x]
```

```
[Out] -1/24*(48*(-a^2 + b^2)^(3/2)*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]] +
 Sec[x]^3*(9*b*(3*a^2 - 2*b^2)*Cos[x]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x
/2] + Sin[x/2]]) + 3*b*(3*a^2 - 2*b^2)*Cos[3*x]*(Log[Cos[x/2] - Sin[x/2]] -
 Log[Cos[x/2] + Sin[x/2]]) + 2*a*(-3*b^2*Sin[x] + 3*a*b*Sin[2*x] + (4*a^2 -
 3*b^2)*Sin[3*x])))/a^4
```

fricas [A] time = 1.21, size = 332, normalized size = 2.94

$$\left[-\frac{6\left(a^2-b^2\right)\sqrt{-a^2+b^2}\,\cos(x)^3\log\left(\frac{2\,ab\cos(x)+\left(2\,a^2-b^2\right)\cos(x)^2+2\,\sqrt{-a^2+b^2}\,(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)-3\left(3\,a^2b-2\,b^3\right)\cos}{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(a^2 - b^2)*sqrt(-a^2 + b^2)*cos(x)^3*log((2*a*b*cos(x) + (2*a^2
- b^2)*cos(x)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(
b^2*cos(x)^2 + 2*a*b*cos(x) + a^2)) - 3*(3*a^2*b - 2*b^3)*cos(x)^3*log(sin(
x) + 1) + 3*(3*a^2*b - 2*b^3)*cos(x)^3*log(-sin(x) + 1) + 2*(3*a^2*b*cos(x)
 - 2*a^3 + 2*(4*a^3 - 3*a*b^2)*cos(x)^2)*sin(x))/(a^4*cos(x)^3), 1/12*(12*(
a^2 - b^2)^(3/2)*arctan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x)))*cos(x)^3
+ 3*(3*a^2*b - 2*b^3)*cos(x)^3*log(sin(x) + 1) - 3*(3*a^2*b - 2*b^3)*cos(x)
^3*log(-sin(x) + 1) - 2*(3*a^2*b*cos(x) - 2*a^3 + 2*(4*a^3 - 3*a*b^2)*cos(x
)^2)*sin(x))/(a^4*cos(x)^3)]
```

giac [B] time = 0.52, size = 226, normalized size = 2.00

$$\frac{\left(3\,a^{2}b-2\,b^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)}{2\,a^{4}}-\frac{\left(3\,a^{2}b-2\,b^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)}{2\,a^{4}}-\frac{2\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-\right.}{\sqrt{a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] 1/2*(3*a^2*b - 2*b^3)*log(abs(tan(1/2*x) + 1))/a^4 - 1/2*(3*a^2*b - 2*b^3)*
log(abs(tan(1/2*x) - 1))/a^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*(pi*floor(1/2*x/pi
 + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x))/sqrt(a^2 -
b^2)))/(sqrt(a^2 - b^2)*a^4) + 1/3*(6*a^2*tan(1/2*x)^5 - 3*a*b*tan(1/2*x)^5
 - 6*b^2*tan(1/2*x)^5 - 20*a^2*tan(1/2*x)^3 + 12*b^2*tan(1/2*x)^3 + 6*a^2*t
an(1/2*x) + 3*a*b*tan(1/2*x) - 6*b^2*tan(1/2*x))/((tan(1/2*x)^2 - 1)^3*a^3)
```

maple [B] time = 0.06, size = 338, normalized size = 2.99

$$\frac{2\arctan\left(\frac{\tan\left(\frac{x}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)\,(a+b)}}-\frac{4\arctan\left(\frac{\tan\left(\frac{x}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)b^{2}}{a^{2}\sqrt{(a-b)\,(a+b)}}+\frac{2\arctan\left(\frac{\tan\left(\frac{x}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)b^{4}}{a^{4}\sqrt{(a-b)\,(a+b)}}-\frac{1}{3a\left(\tan\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{2a\left(\tan\left(\frac{x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^4/(a+b*cos(x)),x)
```

```
[Out] 2/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*x)*(a-b)/((a-b)*(a+b))^(1/2))-4/a^2/((
a-b)*(a+b))^(1/2)*arctan(tan(1/2*x)*(a-b)/((a-b)*(a+b))^(1/2))*b^2+2/a^4/((
a-b)*(a+b))^(1/2)*arctan(tan(1/2*x)*(a-b)/((a-b)*(a+b))^(1/2))*b^4-1/3/a/(t
an(1/2*x)-1)^3-1/2/a/(tan(1/2*x)-1)^2-1/2/a^2/(tan(1/2*x)-1)^2*b+1/a/(tan(1
/2*x)-1)-1/2/a^2/(tan(1/2*x)-1)*b-1/a^3/(tan(1/2*x)-1)*b^2-3/2*b/a^2*ln(tan
(1/2*x)-1)+b^3/a^4*ln(tan(1/2*x)-1)-1/3/a/(tan(1/2*x)+1)^3+1/2/a/(tan(1/2*x
```

```
)+1)^2+1/2/a^2/(tan(1/2*x)+1)^2*b+1/a/(tan(1/2*x)+1)-1/2/a^2/(tan(1/2*x)+1)
*b-1/a^3/(tan(1/2*x)+1)*b^2+3/2*b/a^2*ln(tan(1/2*x)+1)-b^3/a^4*ln(tan(1/2*x
)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 1.10, size = 1666, normalized size = 14.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^4/(a + b*cos(x)),x)
```

```
[Out] (2*atanh((64*tan(x/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(112*a*b^2
 + 128*a^2*b - 64*a^3 - 352*b^3 + (16*b^4)/a + (320*b^5)/a^2 - (112*b^6)/a^
3 - (96*b^7)/a^4 + (48*b^8)/a^5) + (144*b^2*tan(x/2)*(b^6 - a^6 - 3*a^2*b^4
 + 3*a^4*b^2)^(1/2))/(16*a*b^4 + 128*a^4*b - 64*a^5 + 320*b^5 - 352*a^2*b^3
 + 112*a^3*b^2 - (112*b^6)/a - (96*b^7)/a^2 + (48*b^8)/a^3) + (80*b^3*tan(x
/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(320*a*b^5 + 128*a^5*b - 64*
a^6 - 112*b^6 + 16*a^2*b^4 - 352*a^3*b^3 + 112*a^4*b^2 - (96*b^7)/a + (48*b
^8)/a^2) - (144*b^4*tan(x/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(12
8*a^6*b - 112*a*b^6 - 64*a^7 - 96*b^7 + 320*a^2*b^5 + 16*a^3*b^4 - 352*a^4*
b^3 + 112*a^5*b^2 + (48*b^8)/a) + (48*b^5*tan(x/2)*(b^6 - a^6 - 3*a^2*b^4 +
 3*a^4*b^2)^(1/2))/(128*a^7*b - 96*a*b^7 - 64*a^8 + 48*b^8 - 112*a^2*b^6 +
320*a^3*b^5 + 16*a^4*b^4 - 352*a^5*b^3 + 112*a^6*b^2) - (192*b*tan(x/2)*(b^
6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(128*a^3*b - 352*a*b^3 - 64*a^4 + 1
6*b^4 + 112*a^2*b^2 + (320*b^5)/a - (112*b^6)/a^2 - (96*b^7)/a^3 + (48*b^8)
/a^4))*(-(a + b)^3*(a - b)^3)^(1/2))/a^4 - ((4*tan(x/2)^3*(5*a^2 - 3*b^2))/
(3*a^3) - (tan(x/2)*(a*b + 2*a^2 - 2*b^2))/a^3 + (tan(x/2)^5*(a*b - 2*a^2 +
 2*b^2))/a^3)/(3*tan(x/2)^2 - 3*tan(x/2)^4 + tan(x/2)^6 - 1) - (atan(((((3*
a^2*b)/2 - b^3)*((((3*a^2*b)/2 - b^3)*((8*(2*a^12*b - 4*a^13 + 4*a^8*b^5 -
6*a^9*b^4 - 6*a^10*b^3 + 10*a^11*b^2))/a^9 - (8*tan(x/2)*((3*a^2*b)/2 - b^3
)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2))/a^10))/a^4 - (8*tan(x/2)*(16*a*b^8 -
 4*a^8*b + 4*a^9 - 8*b^9 + 16*a^2*b^7 - 48*a^3*b^6 + 3*a^4*b^5 + 39*a^5*b^4
 - 11*a^6*b^3 - 7*a^7*b^2))/a^6)*1i)/a^4 - (((3*a^2*b)/2 - b^3)*((((3*a^2*b
```

```
)/2 - b^3)*((8*(2*a^12*b - 4*a^13 + 4*a^8*b^5 - 6*a^9*b^4 - 6*a^10*b^3 + 10
*a^11*b^2))/a^9 + (8*tan(x/2)*((3*a^2*b)/2 - b^3)*(8*a^10*b + 8*a^8*b^3 - 1
6*a^9*b^2))/a^10))/a^4 + (8*tan(x/2)*(16*a*b^8 - 4*a^8*b + 4*a^9 - 8*b^9 +
16*a^2*b^7 - 48*a^3*b^6 + 3*a^4*b^5 + 39*a^5*b^4 - 11*a^6*b^3 - 7*a^7*b^2))
/a^6)*1i)/a^4)/((16*(6*a*b^10 - 6*a^10*b - 4*b^11 + 18*a^2*b^9 - 31*a^3*b^8
 - 26*a^4*b^7 + 59*a^5*b^6 + 8*a^6*b^5 - 49*a^7*b^4 + 10*a^8*b^3 + 15*a^9*b
^2))/a^9 + (((3*a^2*b)/2 - b^3)*((((3*a^2*b)/2 - b^3)*((8*(2*a^12*b - 4*a^1
3 + 4*a^8*b^5 - 6*a^9*b^4 - 6*a^10*b^3 + 10*a^11*b^2))/a^9 - (8*tan(x/2)*((
3*a^2*b)/2 - b^3)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2))/a^10))/a^4 - (8*tan(
x/2)*(16*a*b^8 - 4*a^8*b + 4*a^9 - 8*b^9 + 16*a^2*b^7 - 48*a^3*b^6 + 3*a^4*
b^5 + 39*a^5*b^4 - 11*a^6*b^3 - 7*a^7*b^2))/a^6))/a^4 + (((3*a^2*b)/2 - b^3
)*((((3*a^2*b)/2 - b^3)*((8*(2*a^12*b - 4*a^13 + 4*a^8*b^5 - 6*a^9*b^4 - 6*
a^10*b^3 + 10*a^11*b^2))/a^9 + (8*tan(x/2)*((3*a^2*b)/2 - b^3)*(8*a^10*b +
8*a^8*b^3 - 16*a^9*b^2))/a^10))/a^4 + (8*tan(x/2)*(16*a*b^8 - 4*a^8*b + 4*a
^9 - 8*b^9 + 16*a^2*b^7 - 48*a^3*b^6 + 3*a^4*b^5 + 39*a^5*b^4 - 11*a^6*b^3
- 7*a^7*b^2))/a^6))/a^4))*((3*a^2*b)/2 - b^3)*2i)/a^4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4(x)}{a + b\cos(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**4/(a+b*cos(x)),x)
```

```
[Out] Integral(tan(x)**4/(a + b*cos(x)), x)
```

3.11 $\int \frac{\tan^3(x)}{a+b\cos(x)}\,dx$

Optimal. Leaf size=57

$$-\frac{b\sec(x)}{a^2}+\frac{(a^2-b^2)\log(\cos(x))}{a^3}-\frac{(a^2-b^2)\log(a+b\cos(x))}{a^3}+\frac{\sec^2(x)}{2a}$$

```
[Out] (a^2-b^2)*ln(cos(x))/a^3-(a^2-b^2)*ln(a+b*cos(x))/a^3-b*sec(x)/a^2+1/2*sec(
x)^2/a
```

Rubi [A] time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2721, 894}

$$\frac{(a^2-b^2)\log(\cos(x))}{a^3}-\frac{(a^2-b^2)\log(a+b\cos(x))}{a^3}-\frac{b\sec(x)}{a^2}+\frac{\sec^2(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3/(a + b*Cos[x]),x]
```

```
[Out] ((a^2 - b^2)*Log[Cos[x]])/a^3 - ((a^2 - b^2)*Log[a + b*Cos[x]])/a^3 - (b*Se
c[x])/a^2 + Sec[x]^2/(2*a)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^3(x)}{a+b\cos(x)}\,dx &= -\mathrm{Subst}\left(\int \frac{b^2-x^2}{x^3(a+x)}\,dx, x, b\cos(x)\right)\\ &= -\mathrm{Subst}\left(\int \left(\frac{b^2}{ax^3}-\frac{b^2}{a^2x^2}+\frac{-a^2+b^2}{a^3x}+\frac{a^2-b^2}{a^3(a+x)}\right)dx, x, b\cos(x)\right)\\ &= \frac{\left(a^2-b^2\right)\log(\cos(x))}{a^3}-\frac{\left(a^2-b^2\right)\log(a+b\cos(x))}{a^3}-\frac{b\sec(x)}{a^2}+\frac{\sec^2(x)}{2a}\end{aligned}$$

Mathematica [A] time = 0.10, size = 46, normalized size = 0.81

$$\frac{2\left(a^2-b^2\right)(\log(\cos(x))-\log(a+b\cos(x)))+a^2\sec^2(x)-2ab\sec(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3/(a + b*Cos[x]),x]
```

```
[Out] (2*(a^2 - b^2)*(Log[Cos[x]] - Log[a + b*Cos[x]]) - 2*a*b*Sec[x] + a^2*Sec[x
]^2)/(2*a^3)
```

fricas [A] time = 0.93, size = 66, normalized size = 1.16

$$-\frac{2\left(a^2-b^2\right)\cos(x)^2\log\left(-b\cos(x)-a\right)-2\left(a^2-b^2\right)\cos(x)^2\log\left(-\cos(x)\right)+2\,ab\cos(x)-a^2}{2\,a^3\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^2 - b^2)*cos(x)^2*log(-b*cos(x) - a) - 2*(a^2 - b^2)*cos(x)^2*lo
g(-cos(x)) + 2*a*b*cos(x) - a^2)/(a^3*cos(x)^2)
```

giac [A] time = 0.51, size = 66, normalized size = 1.16

$$\frac{\left(a^2-b^2\right)\log\left(|\cos(x)|\right)}{a^3}-\frac{\left(a^2b-b^3\right)\log\left(|b\cos(x)+a|\right)}{a^3b}-\frac{2\,ab\cos(x)-a^2}{2\,a^3\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] (a^2 - b^2)*log(abs(cos(x)))/a^3 - (a^2*b - b^3)*log(abs(b*cos(x) + a))/(a^
3*b) - 1/2*(2*a*b*cos(x) - a^2)/(a^3*cos(x)^2)
```

maple [A] time = 0.05, size = 65, normalized size = 1.14

$$-\frac{\ln \left(a+b\cos(x)\right)}{a}+\frac{\ln \left(a+b\cos(x)\right) b^{2}}{a^{3}}-\frac{b}{a^{2}\cos(x)}+\frac{\ln \left(\cos(x)\right)}{a}-\frac{\ln \left(\cos(x)\right) b^{2}}{a^{3}}+\frac{1}{2a\cos(x)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a+b*cos(x)),x)
```

```
[Out] -ln(a+b*cos(x))/a+1/a^3*ln(a+b*cos(x))*b^2-1/a^2*b/cos(x)+ln(cos(x))/a-1/a^
3*ln(cos(x))*b^2+1/2/a/cos(x)^2
```

maxima [A] time = 0.50, size = 56, normalized size = 0.98

$$-\frac{\left(a^{2}-b^{2}\right)\log \left(b\cos(x)+a\right)}{a^{3}}+\frac{\left(a^{2}-b^{2}\right)\log \left(\cos(x)\right)}{a^{3}}-\frac{2\,b\cos(x)-a}{2\,a^{2}\cos(x)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] -(a^2 - b^2)*log(b*cos(x) + a)/a^3 + (a^2 - b^2)*log(cos(x))/a^3 - 1/2*(2*b
*cos(x) - a)/(a^2*cos(x)^2)
```

mupad [B] time = 0.60, size = 115, normalized size = 2.02

$$-\frac{2\,a^{2}\,\mathrm{atanh}\left(\frac{a\tan\left(\frac{x}{2}\right)^{2}}{-b\tan\left(\frac{x}{2}\right)^{2}+a+b}\right)-2\,b^{2}\,\mathrm{atanh}\left(\frac{a\tan\left(\frac{x}{2}\right)^{2}}{-b\tan\left(\frac{x}{2}\right)^{2}+a+b}\right)}{a^{3}}-\frac{2\,a\,b-\tan\left(\frac{x}{2}\right)^{2}\,\left(2\,a^{2}+2\,b\,a\right)}{a^{3}\tan\left(\frac{x}{2}\right)^{4}-2\,a^{3}\tan\left(\frac{x}{2}\right)^{2}+a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3/(a + b*cos(x)),x)
```

```
[Out] - (2*a^2*atanh((a*tan(x/2)^2)/(a + b - b*tan(x/2)^2)) - 2*b^2*atanh((a*tan(
x/2)^2)/(a + b - b*tan(x/2)^2)))/a^3 - (2*a*b - tan(x/2)^2*(2*a*b + 2*a^2))
/(a^3 - 2*a^3*tan(x/2)^2 + a^3*tan(x/2)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{3}\left(x\right)}{a+b\cos\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3/(a+b*cos(x)),x)
```

```
[Out] Integral(tan(x)**3/(a + b*cos(x)), x)
```

3.12 $\int \frac{\tan^2(x)}{a+b\cos(x)}\,dx$

Optimal. Leaf size=61

$$-\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2}-\frac{b\tanh^{-1}(\sin(x))}{a^2}+\frac{\tan(x)}{a}$$

```
[Out] -b*arctanh(sin(x))/a^2-2*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))*(a-b)^(
1/2)*(a+b)^(1/2)/a^2+tan(x)/a
```

Rubi [A] time = 0.22, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2723, 3056, 3001, 3770, 2659, 205}

$$-\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2}-\frac{b\tanh^{-1}(\sin(x))}{a^2}+\frac{\tan(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2/(a + b*Cos[x]),x]
```

```
[Out] (-2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/a^2
 - (b*ArcTanh[Sin[x]])/a^2 + Tan[x]/a
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^2(x)}{a+b\cos(x)}\,dx &= \int \frac{\left(1-\cos^2(x)\right)\sec^2(x)}{a+b\cos(x)}\,dx \\
&= \frac{\tan(x)}{a} + \frac{\int \frac{(-b-a\cos(x))\sec(x)}{a+b\cos(x)}\,dx}{a} \\
&= \frac{\tan(x)}{a} - \frac{b\int \sec(x)\,dx}{a^2} + \frac{\left(-a^2+b^2\right)\int \frac{1}{a+b\cos(x)}\,dx}{a^2} \\
&= -\frac{b\tanh^{-1}(\sin(x))}{a^2} + \frac{\tan(x)}{a} + \frac{\left(2\left(-a^2+b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{a^2} \\
&= -\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2} - \frac{b\tanh^{-1}(\sin(x))}{a^2} + \frac{\tan(x)}{a}
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 85, normalized size = 1.39

$$\frac{-2\sqrt{b^2-a^2}\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)+a\tan(x)+b\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2/(a + b*Cos[x]),x]
```

```
[Out] (-2*Sqrt[-a^2 + b^2]*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]] + b*(Log[
Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) + a*Tan[x])/a^2
```

fricas [A] time = 0.95, size = 203, normalized size = 3.33

$$\left[-\frac{b\cos(x)\log(\sin(x)+1)-b\cos(x)\log(-\sin(x)+1)-\sqrt{-a^2+b^2}\cos(x)\log\left(\frac{2ab\cos(x)+\left(2a^2-b^2\right)\cos(x)^2+2\sqrt{-a^2+\cdots}}{b^2\cos(x)^2+2ab\cos\cdots}\right)\cdots}{2a^2\cos(x)}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(b*cos(x)*log(sin(x) + 1) - b*cos(x)*log(-sin(x) + 1) - sqrt(-a^2 + b
^2)*cos(x)*log((2*a*b*cos(x) + (2*a^2 - b^2)*cos(x)^2 + 2*sqrt(-a^2 + b^2)*
(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*cos(x)^2 + 2*a*b*cos(x) + a^2)) -
 2*a*sin(x))/(a^2*cos(x)), -1/2*(b*cos(x)*log(sin(x) + 1) - b*cos(x)*log(-s
in(x) + 1) + 2*sqrt(a^2 - b^2)*arctan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(
x)))*cos(x) - 2*a*sin(x))/(a^2*cos(x))]
```

giac [B] time = 0.48, size = 111, normalized size = 1.82

$$-\frac{b\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)}{a^2}+\frac{b\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)}{a^2}+\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2}}\right)\right)\cdots}{a^2}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] -b*log(abs(tan(1/2*x) + 1))/a^2 + b*log(abs(tan(1/2*x) - 1))/a^2 + 2*(pi*fl
oor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x))
/sqrt(a^2 - b^2)))*sqrt(a^2 - b^2)/a^2 - 2*tan(1/2*x)/((tan(1/2*x)^2 - 1)*a
)
```

maple [B] time = 0.05, size = 129, normalized size = 2.11

$$-\frac{2\arctan\left(\frac{\tan\left(\frac{x}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}+\frac{2\arctan\left(\frac{\tan\left(\frac{x}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)b^2}{a^2\sqrt{(a-b)(a+b)}}-\frac{1}{a\left(\tan\left(\frac{x}{2}\right)-1\right)}+\frac{b\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{a^2}-\frac{1}{a\left(\tan\left(\frac{x}{2}\right)+1\right)}-\frac{b\ln\left(\tan\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a+b*cos(x)),x)
```

```
[Out] -2/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*x)*(a-b)/((a-b)*(a+b))^(1/2))+2/a^2/(
(a-b)*(a+b))^(1/2)*arctan(tan(1/2*x)*(a-b)/((a-b)*(a+b))^(1/2))*b^2-1/a/(ta
n(1/2*x)-1)+b/a^2*ln(tan(1/2*x)-1)-1/a/(tan(1/2*x)+1)-b/a^2*ln(tan(1/2*x)+1
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.55, size = 77, normalized size = 1.26

$$\frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)\sqrt{b^2-a^2}}{a\,\cos\left(\frac{x}{2}\right)+b\,\cos\left(\frac{x}{2}\right)}\right)\sqrt{b^2-a^2}}{a^2}-\frac{2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{a^2}+\frac{\sin(x)}{a\,\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2/(a + b*cos(x)),x)
```

```
[Out] (2*atanh((sin(x/2)*(b^2 - a^2)^(1/2))/(a*cos(x/2) + b*cos(x/2)))*(b^2 - a^2
)^(1/2))/a^2 - (2*b*atanh(sin(x/2)/cos(x/2)))/a^2 + sin(x)/(a*cos(x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^2(x)}{a+b\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2/(a+b*cos(x)),x)
[Out] Integral(tan(x)**2/(a + b*cos(x)), x)
```

3.13 $\int \frac{\tan(x)}{a+b\cos(x)}\,dx$

Optimal. Leaf size=20

$$\frac{\log(a+b\cos(x))}{a}-\frac{\log(\cos(x))}{a}$$

```
[Out] -ln(cos(x))/a+ln(a+b*cos(x))/a
```

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2721, 36, 29, 31}

$$\frac{\log(a+b\cos(x))}{a}-\frac{\log(\cos(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(a + b*Cos[x]),x]
```

```
[Out] -(Log[Cos[x]]/a) + Log[a + b*Cos[x]]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan(x)}{a+b\cos(x)}\,dx = -\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\cos(x)\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\cos(x)\right)}{a} + \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\cos(x)\right)}{a}$$
$$= -\frac{\log(\cos(x))}{a} + \frac{\log(a+b\cos(x))}{a}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{\log(a+b\cos(x))}{a} - \frac{\log(\cos(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(a + b*Cos[x]),x]
[Out] -(Log[Cos[x]]/a) + Log[a + b*Cos[x]]/a
```

fricas [A] time = 0.65, size = 22, normalized size = 1.10

$$\frac{\log\left(-b\cos(x)-a\right) - \log\left(-\cos(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)),x, algorithm="fricas")
[Out] (log(-b*cos(x) - a) - log(-cos(x)))/a
```

giac [A] time = 0.42, size = 22, normalized size = 1.10

$$\frac{\log\left(|b\cos(x)+a|\right)}{a} - \frac{\log\left(|\cos(x)|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)),x, algorithm="giac")
[Out] log(abs(b*cos(x) + a))/a - log(abs(cos(x)))/a
```

maple [A] time = 0.04, size = 21, normalized size = 1.05

$$-\frac{\ln\left(\cos(x)\right)}{a} + \frac{\ln\left(a+b\cos(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*cos(x)),x)
```

```
[Out] -ln(cos(x))/a+ln(a+b*cos(x))/a
```

maxima [A] time = 0.34, size = 20, normalized size = 1.00

$$\frac{\log\left(b\cos(x)+a\right)}{a}-\frac{\log\left(\cos(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] log(b*cos(x) + a)/a - log(cos(x))/a
```

mupad [B] time = 0.51, size = 48, normalized size = 2.40

$$\frac{\operatorname{atan}\left(\frac{a\sin\left(\frac{x}{2}\right)^2}{a\cos\left(\frac{x}{2}\right)^2 1\mathrm{i}+b\cos\left(\frac{x}{2}\right)^2 1\mathrm{i}-b\sin\left(\frac{x}{2}\right)^2 1\mathrm{i}}\right)2\mathrm{i}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*cos(x)),x)
```

```
[Out] (atan((a*sin(x/2)^2)/(a*cos(x/2)^2*1i + b*cos(x/2)^2*1i - b*sin(x/2)^2*1i))
*2i)/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan\left(x\right)}{a+b\cos\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x)),x)
```

```
[Out] Integral(tan(x)/(a + b*cos(x)), x)
```

3.14 $\int \frac{\cot(x)}{a+b\cos(x)}\,dx$

Optimal. Leaf size=54

$$-\frac{a\log(a+b\cos(x))}{a^2-b^2}+\frac{\log(1-\cos(x))}{2(a+b)}+\frac{\log(\cos(x)+1)}{2(a-b)}$$

```
[Out] 1/2*ln(1-cos(x))/(a+b)+1/2*ln(1+cos(x))/(a-b)-a*ln(a+b*cos(x))/(a^2-b^2)
```

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2721, 801}

$$-\frac{a\log(a+b\cos(x))}{a^2-b^2}+\frac{\log(1-\cos(x))}{2(a+b)}+\frac{\log(\cos(x)+1)}{2(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a + b*Cos[x]),x]
```

```
[Out] Log[1 - Cos[x]]/(2*(a + b)) + Log[1 + Cos[x]]/(2*(a - b)) - (a*Log[a + b*Co
s[x]])/(a^2 - b^2)
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot(x)}{a+b\cos(x)}\,dx = -\text{Subst}\left(\int \frac{x}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\cos(x)\right)$$
$$= -\text{Subst}\left(\int \left(\frac{1}{2(a+b)(b-x)} + \frac{a}{(a-b)(a+b)(a+x)} - \frac{1}{2(a-b)(b+x)}\right)dx, x, b\cos(x)\right)$$
$$= \frac{\log(1-\cos(x))}{2(a+b)} + \frac{\log(1+\cos(x))}{2(a-b)} - \frac{a\log(a+b\cos(x))}{a^2-b^2}$$

Mathematica [A] time = 0.07, size = 50, normalized size = 0.93

$$-\frac{a\log(a+b\cos(x))}{a^2-b^2} + \frac{\log\left(\sin\left(\frac{x}{2}\right)\right)}{a+b} + \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{a-b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a + b*Cos[x]),x]
```

```
[Out] Log[Cos[x/2]]/(a - b) - (a*Log[a + b*Cos[x]])/(a^2 - b^2) + Log[Sin[x/2]]/(
a + b)
```

fricas [A] time = 0.93, size = 53, normalized size = 0.98

$$-\frac{2\,a\log\left(-b\cos(x)-a\right)-(a+b)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(a-b)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)}{2\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*log(-b*cos(x) - a) - (a + b)*log(1/2*cos(x) + 1/2) - (a - b)*log(
-1/2*cos(x) + 1/2))/(a^2 - b^2)
```

giac [A] time = 0.43, size = 54, normalized size = 1.00

$$-\frac{ab\log\left(|b\cos(x)+a|\right)}{a^2b-b^3} + \frac{\log\left(\cos(x)+1\right)}{2\left(a-b\right)} + \frac{\log\left(-\cos(x)+1\right)}{2\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] -a*b*log(abs(b*cos(x) + a))/(a^2*b - b^3) + 1/2*log(cos(x) + 1)/(a - b) + 1
/2*log(-cos(x) + 1)/(a + b)
```

maple [A] time = 0.05, size = 54, normalized size = 1.00

$$-\frac{a \ln \left(a+b \cos (x)\right)}{\left(a+b\right) \left(a-b\right)}+\frac{\ln \left(-1+\cos (x)\right)}{2 a+2 b}+\frac{\ln \left(\cos (x)+1\right)}{2 a-2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*cos(x)),x)
```

```
[Out] -a/(a+b)/(a-b)*ln(a+b*cos(x))+1/(2*a+2*b)*ln(-1+cos(x))+1/(2*a-2*b)*ln(cos(
x)+1)
```

maxima [A] time = 0.66, size = 48, normalized size = 0.89

$$-\frac{a \log \left(b \cos (x)+a\right)}{a^{2}-b^{2}}+\frac{\log \left(\cos (x)+1\right)}{2 \left(a-b\right)}+\frac{\log \left(\cos (x)-1\right)}{2 \left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] -a*log(b*cos(x) + a)/(a^2 - b^2) + 1/2*log(cos(x) + 1)/(a - b) + 1/2*log(co
s(x) - 1)/(a + b)
```

mupad [B] time = 0.50, size = 47, normalized size = 0.87

$$\frac{\ln \left(\tan \left(\frac{x}{2}\right)\right)}{a+b}-\frac{a\,\ln \left(a+b+a\,\tan \left(\frac{x}{2}\right)^{2}-b\,\tan \left(\frac{x}{2}\right)^{2}\right)}{a^{2}-b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*cos(x)),x)
```

```
[Out] log(tan(x/2))/(a + b) - (a*log(a + b + a*tan(x/2)^2 - b*tan(x/2)^2))/(a^2 -
 b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot (x)}{a+b \cos (x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*cos(x)),x)
```

```
[Out] Integral(cot(x)/(a + b*cos(x)), x)
```

3.15 $\int \frac{\cot^2(x)}{a+b\cos(x)}\,dx$

Optimal. Leaf size=77

$$-\frac{a\cot(x)}{a^2-b^2}+\frac{b\csc(x)}{a^2-b^2}-\frac{2a^2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] -2*a^2*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^(3/2)-a
*cot(x)/(a^2-b^2)+b*csc(x)/(a^2-b^2)
```

Rubi [A] time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2727, 3767, 8, 2606, 2659, 205}

$$-\frac{a\cot(x)}{a^2-b^2}+\frac{b\csc(x)}{a^2-b^2}-\frac{2a^2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2/(a + b*Cos[x]),x]
```

```
[Out] (-2*a^2*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a + b)^
(3/2)) - (a*Cot[x])/(a^2 - b^2) + (b*Csc[x])/(a^2 - b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2727

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[(b*g)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[(a^2*g^2)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^2(x)}{a+b\cos(x)}\,dx &= \frac{a\int \csc^2(x)\,dx}{a^2-b^2} - \frac{a^2\int \frac{1}{a+b\cos(x)}\,dx}{a^2-b^2} - \frac{b\int \cot(x)\csc(x)\,dx}{a^2-b^2}\\
&= -\frac{a\,\mathrm{Subst}(\int 1\,dx, x, \cot(x))}{a^2-b^2} - \frac{\left(2a^2\right)\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)}{a^2-b^2} + \frac{b\,\mathrm{Subst}(\int 1\,dx, x}{a^2-b^2}\\
&= -\frac{2a^2\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}} - \frac{a\cot(x)}{a^2-b^2} + \frac{b\csc(x)}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.33, size = 67, normalized size = 0.87

$$\frac{b\csc(x)-a\cot(x)}{a^2-b^2} - \frac{2a^2\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2/(a + b*Cos[x]),x]
```

```
[Out] (-2*a^2*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) +
(-(a*Cot[x]) + b*Csc[x])/(a^2 - b^2)
```

fricas [A] time = 0.85, size = 230, normalized size = 2.99

$$\left[\frac{\sqrt{-a^2+b^2}\,a^2\log\left(\frac{2\,ab\cos(x)+\left(2\,a^2-b^2\right)\cos(x)^2+2\,\sqrt{-a^2+b^2}\,(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)\sin(x)+2\,a^2b-2\,b^3-2\left(a^3-ab^2\right)\cos\right.}{2\left(a^4-2\,a^2b^2+b^4\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*a^2*log((2*a*b*cos(x) + (2*a^2 - b^2)*cos(x)^2 + 2*s
qrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*cos(x)^2 + 2*a*b*
cos(x) + a^2))*sin(x) + 2*a^2*b - 2*b^3 - 2*(a^3 - a*b^2)*cos(x))/((a^4 - 2
*a^2*b^2 + b^4)*sin(x)), -(sqrt(a^2 - b^2)*a^2*arctan(-(a*cos(x) + b)/(sqrt
(a^2 - b^2)*sin(x)))*sin(x) - a^2*b + b^3 + (a^3 - a*b^2)*cos(x))/((a^4 - 2
*a^2*b^2 + b^4)*sin(x))]
```

giac [A] time = 0.49, size = 91, normalized size = 1.18

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,x\right)-b\tan\left(\frac{1}{2}\,x\right)}{\sqrt{a^2-b^2}}\right)\right)a^2}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{\tan\left(\frac{1}{2}\,x\right)}{2\,(a-b)}-\frac{1}{2\,(a+b)\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan
(1/2*x))/sqrt(a^2 - b^2)))*a^2/(a^2 - b^2)^(3/2) + 1/2*tan(1/2*x)/(a - b) -
 1/2/((a + b)*tan(1/2*x))
```

maple [A] time = 0.06, size = 78, normalized size = 1.01

$$\frac{\tan\left(\frac{x}{2}\right)}{2a-2b}-\frac{2a^2\arctan\left(\frac{\tan\left(\frac{x}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)\,(a+b)\,\sqrt{(a-b)\,(a+b)}}-\frac{1}{2\,(a+b)\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a+b*cos(x)),x)
```

```
[Out] 1/2/(a-b)*tan(1/2*x)-2/(a-b)/(a+b)*a^2/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*x
)*(a-b)/((a-b)*(a+b))^(1/2))-1/2/(a+b)/tan(1/2*x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.54, size = 86, normalized size = 1.12

$$\frac{\tan\left(\frac{x}{2}\right)}{2\,a-2\,b}-\frac{2\,a^2\,\mathrm{atan}\left(\frac{\tan\left(\frac{x}{2}\right)\,\left(a^2-b^2\right)}{{\left(a+b\right)}^{3/2}\,\sqrt{a-b}}\right)}{{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}-\frac{a-b}{\tan\left(\frac{x}{2}\right)\,\left(a+b\right)\,\left(2\,a-2\,b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2/(a + b*cos(x)),x)
```

```
[Out] tan(x/2)/(2*a - 2*b) - (2*a^2*atan((tan(x/2)*(a^2 - b^2))/((a + b)^(3/2)*(a
 - b)^(1/2))))/((a + b)^(3/2)*(a - b)^(3/2)) - (a - b)/(tan(x/2)*(a + b)*(2
*a - 2*b))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(x)}{a+b\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2/(a+b*cos(x)),x)
```

```
[Out] Integral(cot(x)**2/(a + b*cos(x)), x)
```

3.16 $\int \frac{\cot^3(x)}{a+b\cos(x)} \, dx$

Optimal. Leaf size=93

$$-\frac{\csc^2(x)(a-b\cos(x))}{2(a^2-b^2)}+\frac{a^3\log(a+b\cos(x))}{(a^2-b^2)^2}-\frac{(2a+b)\log(1-\cos(x))}{4(a+b)^2}-\frac{(2a-b)\log(\cos(x)+1)}{4(a-b)^2}$$

```
[Out] -1/2*(a-b*cos(x))*csc(x)^2/(a^2-b^2)-1/4*(2*a+b)*ln(1-cos(x))/(a+b)^2-1/4*(
2*a-b)*ln(1+cos(x))/(a-b)^2+a^3*ln(a+b*cos(x))/(a^2-b^2)^2
```

Rubi [A] time = 0.18, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2721, 1647, 801}

$$\frac{a^3\log(a+b\cos(x))}{(a^2-b^2)^2}-\frac{\csc^2(x)(a-b\cos(x))}{2(a^2-b^2)}-\frac{(2a+b)\log(1-\cos(x))}{4(a+b)^2}-\frac{(2a-b)\log(\cos(x)+1)}{4(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3/(a + b*Cos[x]),x]
```

```
[Out] -((a - b*Cos[x])*Csc[x]^2)/(2*(a^2 - b^2)) - ((2*a + b)*Log[1 - Cos[x]])/(4
*(a + b)^2) - ((2*a - b)*Log[1 + Cos[x]])/(4*(a - b)^2) + (a^3*Log[a + b*Co
s[x]])/(a^2 - b^2)^2
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(x)}{a+b\cos(x)}\,dx = -\text{Subst}\left(\int \frac{x^3}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\cos(x)\right)$$

$$= -\frac{(a-b\cos(x))\csc^2(x)}{2\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{ab^4}{a^2-b^2}-\frac{b^2\left(2a^2-b^2\right)x}{a^2-b^2}}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\cos(x)\right)}{2b^2}$$

$$= -\frac{(a-b\cos(x))\csc^2(x)}{2\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\int \left(-\frac{b^2(2a+b)}{2(a+b)^2(b-x)} - \frac{2a^3b^2}{(a-b)^2(a+b)^2(a+x)} + \frac{(2a-b)b^2}{2(a-b)^2(b+x)}\right) dx, x, b\cos\right.}{2b^2}$$

$$= -\frac{(a-b\cos(x))\csc^2(x)}{2\left(a^2-b^2\right)} - \frac{(2a+b)\log(1-\cos(x))}{4(a+b)^2} - \frac{(2a-b)\log(1+\cos(x))}{4(a-b)^2} + \frac{a^3\log(a+}{\left(a^2-\right.}$$

Mathematica [A] time = 0.57, size = 100, normalized size = 1.08

$$\frac{1}{8}\left(\frac{8a^3\log(a+b\cos(x))}{\left(a^2-b^2\right)^2} - \frac{\csc^2\left(\frac{x}{2}\right)}{a+b} - \frac{\sec^2\left(\frac{x}{2}\right)}{a-b} - \frac{4(2a+b)\log\left(\sin\left(\frac{x}{2}\right)\right)}{(a+b)^2} + \frac{4(b-2a)\log\left(\cos\left(\frac{x}{2}\right)\right)}{(a-b)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3/(a + b*Cos[x]),x]
```

```
[Out] (-(Csc[x/2]^2/(a + b)) + (4*(-2*a + b)*Log[Cos[x/2]])/(a - b)^2 + (8*a^3*Lo
g[a + b*Cos[x]])/(a^2 - b^2)^2 - (4*(2*a + b)*Log[Sin[x/2]])/(a + b)^2 - Se
c[x/2]^2/(a - b))/8
```

fricas [B] time = 0.59, size = 185, normalized size = 1.99

$$-\frac{2\,a^3-2\,ab^2-2\left(a^2b-b^3\right)\cos(x)+4\left(a^3\cos(x)^2-a^3\right)\log\left(-b\cos(x)-a\right)+\left(2\,a^3+3\,a^2b-b^3-\left(2\,a^3+3\,a^2b\right.\right.}{4\left(a^4-2\,a^2b^2+b^4-\left(a^4-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^3 - 2*a*b^2 - 2*(a^2*b - b^3)*cos(x) + 4*(a^3*cos(x)^2 - a^3)*log
(-b*cos(x) - a) + (2*a^3 + 3*a^2*b - b^3 - (2*a^3 + 3*a^2*b - b^3)*cos(x)^2
)*log(1/2*cos(x) + 1/2) + (2*a^3 - 3*a^2*b + b^3 - (2*a^3 - 3*a^2*b + b^3)*
cos(x)^2)*log(-1/2*cos(x) + 1/2))/(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2
 + b^4)*cos(x)^2)
```

giac [A] time = 0.46, size = 138, normalized size = 1.48

$$\frac{a^3 b \log\left(|b\cos(x) + a|\right)}{a^4 b - 2\,a^2 b^3 + b^5} - \frac{(2\,a - b)\log\left(\cos(x) + 1\right)}{4\left(a^2 - 2\,ab + b^2\right)} - \frac{(2\,a + b)\log\left(-\cos(x) + 1\right)}{4\left(a^2 + 2\,ab + b^2\right)} + \frac{a^3 - ab^2 - \left(a^2 b - b^3\right)\cos(x}{2\,(a+b)^2 (a-b)^2 (\cos(x) + 1)(\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] a^3*b*log(abs(b*cos(x) + a))/(a^4*b - 2*a^2*b^3 + b^5) - 1/4*(2*a - b)*log(
cos(x) + 1)/(a^2 - 2*a*b + b^2) - 1/4*(2*a + b)*log(-cos(x) + 1)/(a^2 + 2*a
*b + b^2) + 1/2*(a^3 - a*b^2 - (a^2*b - b^3)*cos(x))/((a + b)^2*(a - b)^2*(
cos(x) + 1)*(cos(x) - 1))
```

maple [A] time = 0.06, size = 114, normalized size = 1.23

$$\frac{a^3 \ln\left(a + b\cos(x)\right)}{(a+b)^2 (a-b)^2} + \frac{1}{(4a + 4b)(-1 + \cos(x))} - \frac{\ln\left(-1 + \cos(x)\right) a}{2\,(a+b)^2} - \frac{\ln\left(-1 + \cos(x)\right) b}{4\,(a+b)^2} - \frac{1}{(4a - 4b)(\cos(x) + 1)} - \ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a+b*cos(x)),x)
```

```
[Out] a^3/(a+b)^2/(a-b)^2*ln(a+b*cos(x))+1/(4*a+4*b)/(-1+cos(x))-1/2/(a+b)^2*ln(-
1+cos(x))*a-1/4/(a+b)^2*ln(-1+cos(x))*b-1/(4*a-4*b)/(cos(x)+1)-1/2/(a-b)^2*
ln(cos(x)+1)*a+1/4/(a-b)^2*ln(cos(x)+1)*b
```

maxima [A] time = 0.87, size = 116, normalized size = 1.25

$$\frac{a^3 \log\left(b\cos(x) + a\right)}{a^4 - 2\,a^2 b^2 + b^4} - \frac{(2\,a - b)\log\left(\cos(x) + 1\right)}{4\left(a^2 - 2\,ab + b^2\right)} - \frac{(2\,a + b)\log\left(\cos(x) - 1\right)}{4\left(a^2 + 2\,ab + b^2\right)} - \frac{b\cos(x) - a}{2\left(\left(a^2 - b^2\right)\cos(x)^2 - a^2 + b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] a^3*log(b*cos(x) + a)/(a^4 - 2*a^2*b^2 + b^4) - 1/4*(2*a - b)*log(cos(x) +
1)/(a^2 - 2*a*b + b^2) - 1/4*(2*a + b)*log(cos(x) - 1)/(a^2 + 2*a*b + b^2)
- 1/2*(b*cos(x) - a)/((a^2 - b^2)*cos(x)^2 - a^2 + b^2)
```

mupad [B] time = 0.58, size = 116, normalized size = 1.25

$$\frac{a^3\ \ln\left(a+b+a\tan\left(\frac{x}{2}\right)^2-b\tan\left(\frac{x}{2}\right)^2\right)}{a^4-2\,a^2\,b^2+b^4}-\frac{\tan\left(\frac{x}{2}\right)^2}{2\,\left(4\,a-4\,b\right)}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)\,\left(2\,a+b\right)}{2\,a^2+4\,a\,b+2\,b^2}-\frac{a-b}{2\tan\left(\frac{x}{2}\right)^2\,\left(a+b\right)\,\left(4\,a-4\,b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a + b*cos(x)),x)
```

```
[Out] (a^3*log(a + b + a*tan(x/2)^2 - b*tan(x/2)^2))/(a^4 + b^4 - 2*a^2*b^2) - ta
n(x/2)^2/(2*(4*a - 4*b)) - (log(tan(x/2))*(2*a + b))/(4*a*b + 2*a^2 + 2*b^2
) - (a - b)/(2*tan(x/2)^2*(a + b)*(4*a - 4*b))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^3\left(x\right)}{a+b\cos\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3/(a+b*cos(x)),x)
```

```
[Out] Integral(cot(x)**3/(a + b*cos(x)), x)
```

3.17 $\int \frac{\cot^4(x)}{a+b\cos(x)}\,dx$

Optimal. Leaf size=138

$$\frac{2a^4\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}-\frac{a\cot^3(x)}{3\left(a^2-b^2\right)}+\frac{b\csc^3(x)}{3\left(a^2-b^2\right)}-\frac{a^2b\csc(x)}{\left(a^2-b^2\right)^2}-\frac{b\csc(x)}{a^2-b^2}+\frac{a^3\cot(x)}{\left(a^2-b^2\right)^2}$$

```
[Out] 2*a^4*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b)^(5/2)+a^
3*cot(x)/(a^2-b^2)^2-1/3*a*cot(x)^3/(a^2-b^2)-a^2*b*csc(x)/(a^2-b^2)^2-b*cs
c(x)/(a^2-b^2)+1/3*b*csc(x)^3/(a^2-b^2)
```

Rubi [A] time = 0.21, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {2727, 2607, 30, 2606, 3767, 8, 2659, 205}

$$-\frac{a\cot^3(x)}{3\left(a^2-b^2\right)}+\frac{a^3\cot(x)}{\left(a^2-b^2\right)^2}+\frac{b\csc^3(x)}{3\left(a^2-b^2\right)}-\frac{a^2b\csc(x)}{\left(a^2-b^2\right)^2}-\frac{b\csc(x)}{a^2-b^2}+\frac{2a^4\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^4/(a + b*Cos[x]),x]
```

```
[Out] (2*a^4*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)^(
5/2)) + (a^3*Cot[x])/(a^2 - b^2)^2 - (a*Cot[x]^3)/(3*(a^2 - b^2)) - (a^2*b*
Csc[x])/(a^2 - b^2)^2 - (b*Csc[x])/(a^2 - b^2) + (b*Csc[x]^3)/(3*(a^2 - b^2
))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2727

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[(b*g)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[(a^2*g^2)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\cot^4(x)}{a+b\cos(x)}\,dx = \frac{a\int \cot^2(x)\csc^2(x)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\cot^2(x)}{a+b\cos(x)}\,dx}{a^2-b^2} - \frac{b\int \cot^3(x)\csc(x)\,dx}{a^2-b^2}$$

$$= -\frac{a^3\int \csc^2(x)\,dx}{\left(a^2-b^2\right)^2} + \frac{a^4\int \frac{1}{a+b\cos(x)}\,dx}{\left(a^2-b^2\right)^2} + \frac{\left(a^2b\right)\int \cot(x)\csc(x)\,dx}{\left(a^2-b^2\right)^2} + \frac{a\,\text{Subst}\left(\int x^2\,dx, x, -\cot(\right.}{a^2-b^2}$$

$$= -\frac{a\cot^3(x)}{3\left(a^2-b^2\right)} - \frac{b\csc(x)}{a^2-b^2} + \frac{b\csc^3(x)}{3\left(a^2-b^2\right)} + \frac{a^3\,\text{Subst}(\int 1\,dx, x, \cot(x))}{\left(a^2-b^2\right)^2} + \frac{\left(2a^4\right)\text{Subst}\left(\int \frac{1}{a+b+(a-}\right.}{\left(a^2-b\right.}$$

$$= \frac{2a^4\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} + \frac{a^3\cot(x)}{\left(a^2-b^2\right)^2} - \frac{a\cot^3(x)}{3\left(a^2-b^2\right)} - \frac{a^2b\csc(x)}{\left(a^2-b^2\right)^2} - \frac{b\csc(x)}{a^2-b^2} + \frac{b\csc^3(x)}{3\left(a^2-b^2\right)}$$

Mathematica [A] time = 0.65, size = 112, normalized size = 0.81

$$-\frac{\csc^3(x)\left(6b\left(b^2-2a^2\right)\cos(2x)+\left(4a^2-b^2\right)(a\cos(3x)+2b)-3ab^2\cos(x)\right)}{12(a-b)^2(a+b)^2} - \frac{2a^4\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^4/(a + b*Cos[x]),x]
```

```
[Out] (-2*a^4*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) -
((-3*a*b^2*Cos[x] + 6*b*(-2*a^2 + b^2)*Cos[2*x] + (4*a^2 - b^2)*(2*b + a*Co
s[3*x]))*Csc[x]^3)/(12*(a - b)^2*(a + b)^2)
```

fricas [A] time = 0.91, size = 456, normalized size = 3.30

$$\left[-\frac{10\,a^4b - 14\,a^2b^3 + 4\,b^5 + 2\left(4\,a^5 - 5\,a^3b^2 + ab^4\right)\cos(x)^3 - 3\left(a^4\cos(x)^2 - a^4\right)\sqrt{-a^2+b^2}\,\log\left(\frac{2\,ab\cos(x)+\left(2\,a^2-b^2\right)}{}\right.}{6\left(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6 - \left(a^6 - 3\,a^4b^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4/(a+b*cos(x)),x, algorithm="fricas")
```

```
[Out] [-1/6*(10*a^4*b - 14*a^2*b^3 + 4*b^5 + 2*(4*a^5 - 5*a^3*b^2 + a*b^4)*cos(x)
^3 - 3*(a^4*cos(x)^2 - a^4)*sqrt(-a^2 + b^2)*log((2*a*b*cos(x) + (2*a^2 - b
```

```
^2)*cos(x)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2
*cos(x)^2 + 2*a*b*cos(x) + a^2))*sin(x) - 6*(2*a^4*b - 3*a^2*b^3 + b^5)*cos
(x)^2 - 6*(a^5 - a^3*b^2)*cos(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 - (a^
6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(x)^2)*sin(x)), -1/3*(5*a^4*b - 7*a^2*b
^3 + 2*b^5 + (4*a^5 - 5*a^3*b^2 + a*b^4)*cos(x)^3 + 3*(a^4*cos(x)^2 - a^4)*
sqrt(a^2 - b^2)*arctan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x)))*sin(x) - 3
*(2*a^4*b - 3*a^2*b^3 + b^5)*cos(x)^2 - 3*(a^5 - a^3*b^2)*cos(x))/((a^6 - 3
*a^4*b^2 + 3*a^2*b^4 - b^6 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(x)^2)*
sin(x))]
```

giac [A] time = 0.80, size = 210, normalized size = 1.52

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2}}\right)\right)a^4}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{a^2\tan\left(\frac{1}{2}x\right)^3-2ab\tan\left(\frac{1}{2}x\right)^3+b^2\tan\left(\frac{1}{2}x\right.}{24\left(a^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4/(a+b*cos(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*ta
n(1/2*x))/sqrt(a^2 - b^2)))*a^4/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) +
 1/24*(a^2*tan(1/2*x)^3 - 2*a*b*tan(1/2*x)^3 + b^2*tan(1/2*x)^3 - 15*a^2*ta
n(1/2*x) + 24*a*b*tan(1/2*x) - 9*b^2*tan(1/2*x))/(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3) + 1/24*(15*a*tan(1/2*x)^2 + 9*b*tan(1/2*x)^2 - a - b)/((a^2 + 2*a*b +
 b^2)*tan(1/2*x)^3)
```

maple [A] time = 0.06, size = 153, normalized size = 1.11

$$\frac{a\left(\tan^3\left(\frac{x}{2}\right)\right)}{24(a-b)^2}-\frac{\left(\tan^3\left(\frac{x}{2}\right)\right)b}{24(a-b)^2}-\frac{5a\tan\left(\frac{x}{2}\right)}{8(a-b)^2}+\frac{3\tan\left(\frac{x}{2}\right)b}{8(a-b)^2}+\frac{2a^4\arctan\left(\frac{\tan\left(\frac{x}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)^2(a+b)^2\sqrt{(a-b)(a+b)}}-\frac{1}{24(a+b)\tan\left(\frac{x}{2}\right)^3}+\frac{}{8(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4/(a+b*cos(x)),x)
```

```
[Out] 1/24/(a-b)^2*a*tan(1/2*x)^3-1/24/(a-b)^2*tan(1/2*x)^3*b-5/8/(a-b)^2*a*tan(1
/2*x)+3/8/(a-b)^2*tan(1/2*x)*b+2/(a-b)^2/(a+b)^2*a^4/((a-b)*(a+b))^(1/2)*ar
ctan(tan(1/2*x)*(a-b)/((a-b)*(a+b))^(1/2))-1/24/(a+b)/tan(1/2*x)^3+5/8/(a+b
)^2/tan(1/2*x)*a+3/8/(a+b)^2/tan(1/2*x)*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4/(a+b*cos(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more details)Is 4*b^2-4*a^2 positive or negative?
```

mupad [B] time = 0.63, size = 183, normalized size = 1.33

$$\frac{\tan\left(\frac{x}{2}\right)^3}{3\,(8\,a-8\,b)}-\tan\left(\frac{x}{2}\right)\left(\frac{4}{8\,a-8\,b}+\frac{8\,a+8\,b}{(8\,a-8\,b)^2}\right)-\frac{\frac{a^2-2\,a\,b+b^2}{3\,(a+b)}+\frac{\tan\left(\frac{x}{2}\right)^2\left(-5\,a^3+7\,a^2\,b+a\,b^2-3\,b^3\right)}{(a+b)^2}}{\tan\left(\frac{x}{2}\right)^3\left(8\,a^2-16\,a\,b+8\,b^2\right)}+\frac{2\,a^4\,\mathrm{atan}\left(\frac{\tan\left(\frac{x}{2}\right)\left(a^4-2\,a^2\right.}{(a+b)^{5/2}\,(a-b}\right.}{(a+b)^{5/2}\,(a-b)^{5/}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4/(a + b*cos(x)),x)
```

```
[Out] tan(x/2)^3/(3*(8*a - 8*b)) - tan(x/2)*(4/(8*a - 8*b) + (8*a + 8*b)/(8*a - 8
*b)^2) - ((a^2 - 2*a*b + b^2)/(3*(a + b)) + (tan(x/2)^2*(a*b^2 + 7*a^2*b -
5*a^3 - 3*b^3))/(a + b)^2)/(tan(x/2)^3*(8*a^2 - 16*a*b + 8*b^2)) + (2*a^4*a
tan((tan(x/2)*(a^4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(3/2))))/((a
+ b)^(5/2)*(a - b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(x)}{a+b\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**4/(a+b*cos(x)),x)
```

```
[Out] Integral(cot(x)**4/(a + b*cos(x)), x)
```

3.18 $\int \frac{\cot(x)}{\sqrt{3-\cos(x)}}\,dx$

Optimal. Leaf size=44

$$-\frac{1}{2}\tanh^{-1}\left(\frac{1}{2}\sqrt{3-\cos(x)}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3-\cos(x)}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctanh(1/2*(3-cos(x))^(1/2))-1/2*arctanh(1/2*(3-cos(x))^(1/2)*2^(1/2)
)*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2721, 827, 1166, 206}

$$-\frac{1}{2}\tanh^{-1}\left(\frac{1}{2}\sqrt{3-\cos(x)}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3-\cos(x)}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Sqrt[3 - Cos[x]],x]
```

```
[Out] -ArcTanh[Sqrt[3 - Cos[x]]/2]/2 - ArcTanh[Sqrt[3 - Cos[x]]/Sqrt[2]]/Sqrt[2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot(x)}{\sqrt{3-\cos(x)}}\,dx &= -\operatorname{Subst}\left(\int \frac{x}{\sqrt{3+x}\,\left(1-x^2\right)}\,dx, x, -\cos(x)\right)\\
&= -\left(2\operatorname{Subst}\left(\int \frac{-3+x^2}{-8+6x^2-x^4}\,dx, x, \sqrt{3-\cos(x)}\right)\right)\\
&= -\operatorname{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{3-\cos(x)}\right)-\operatorname{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \sqrt{3-\cos(x)}\right)\\
&= -\frac{1}{2}\tanh^{-1}\left(\frac{1}{2}\sqrt{3-\cos(x)}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3-\cos(x)}}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 44, normalized size = 1.00

$$
-\frac{1}{2}\tanh^{-1}\left(\frac{1}{2}\sqrt{3-\cos(x)}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3-\cos(x)}}{\sqrt{2}}\right)}{\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Sqrt[3 - Cos[x]],x]
```

```
[Out] -1/2*ArcTanh[Sqrt[3 - Cos[x]]/2] - ArcTanh[Sqrt[3 - Cos[x]]/Sqrt[2]]/Sqrt[2
]
```

fricas [B] time = 0.86, size = 77, normalized size = 1.75

$$
\frac{1}{8}\sqrt{2}\log\left(\frac{\cos(x)^2+4\left(\sqrt{2}\cos(x)-5\sqrt{2}\right)\sqrt{-\cos(x)+3}-18\cos(x)+49}{\cos(x)^2-2\cos(x)+1}\right)+\frac{1}{4}\log\left(-\frac{4\sqrt{-\cos(x)+3}+\cos(x}{\cos(x)+1}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(3-cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log((cos(x)^2 + 4*(sqrt(2)*cos(x) - 5*sqrt(2))*sqrt(-cos(x) + 3
) - 18*cos(x) + 49)/(cos(x)^2 - 2*cos(x) + 1)) + 1/4*log(-(4*sqrt(-cos(x) +
 3) + cos(x) - 7)/(cos(x) + 1))
```

giac [B] time = 0.69, size = 68, normalized size = 1.55

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sqrt{-\cos(x)+3}\right|}{2\left(\sqrt{2}+\sqrt{-\cos(x)+3}\right)}\right)-\frac{1}{4}\log\left(\sqrt{-\cos(x)+3}+2\right)+\frac{1}{4}\log\left(-\sqrt{-\cos(x)+3}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(3-cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*sqrt(-cos(x) + 3))/(sqrt(2) + sqrt(-
cos(x) + 3))) - 1/4*log(sqrt(-cos(x) + 3) + 2) + 1/4*log(-sqrt(-cos(x) + 3)
 + 2)
```

maple [B] time = 0.49, size = 81, normalized size = 1.84

$$-\frac{\sqrt{2}\ \mathrm{arctanh}\left(\frac{-\sqrt{2}\ \cos\left(\frac{x}{2}\right)+2\sqrt{2}}{\sqrt{2\left(\sin^{2}\left(\frac{x}{2}\right)\right)+2}}\right)}{4}-\frac{\sqrt{2}\ \mathrm{arctanh}\left(\frac{\left(2+\cos\left(\frac{x}{2}\right)\right)\sqrt{2}}{\sqrt{2\left(\sin^{2}\left(\frac{x}{2}\right)\right)+2}}\right)}{4}-\frac{\mathrm{arctanh}\left(\frac{2}{\sqrt{2\left(\sin^{2}\left(\frac{x}{2}\right)\right)+2}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(3-cos(x))^(1/2),x)
```

```
[Out] -1/4*2^(1/2)*arctanh((-2^(1/2)*cos(1/2*x)+2*2^(1/2))/(2*sin(1/2*x)^2+2)^(1/
2))-1/4*2^(1/2)*arctanh(1/(2*sin(1/2*x)^2+2)^(1/2)*(2+cos(1/2*x))*2^(1/2))-
1/2*arctanh(2/(2*sin(1/2*x)^2+2)^(1/2))
```

maxima [A] time = 2.54, size = 63, normalized size = 1.43

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{-\cos(x)+3}}{\sqrt{2}+\sqrt{-\cos(x)+3}}\right)-\frac{1}{4}\log\left(\sqrt{-\cos(x)+3}+2\right)+\frac{1}{4}\log\left(\sqrt{-\cos(x)+3}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(3-cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(-(sqrt(2) - sqrt(-cos(x) + 3))/(sqrt(2) + sqrt(-cos(x) + 3)
)) - 1/4*log(sqrt(-cos(x) + 3) + 2) + 1/4*log(sqrt(-cos(x) + 3) - 2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cot(x)}{\sqrt{3-\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(3 - cos(x))^(1/2),x)
```

```
[Out] int(cot(x)/(3 - cos(x))^(1/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\sqrt{3-\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(3-cos(x))**(1/2),x)
```

```
[Out] Integral(cot(x)/sqrt(3 - cos(x)), x)
```

3.19 $\int \sqrt{a+b\cos(x)}\, \tan(x)\, dx$

Optimal. Leaf size=37

$$2\sqrt{a}\, \tanh^{-1}\left(\frac{\sqrt{a+b\cos(x)}}{\sqrt{a}}\right)-2\sqrt{a+b\cos(x)}$$

```
[Out] 2*arctanh((a+b*cos(x))^(1/2)/a^(1/2))*a^(1/2)-2*(a+b*cos(x))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2721, 50, 63, 207}

$$2\sqrt{a}\, \tanh^{-1}\left(\frac{\sqrt{a+b\cos(x)}}{\sqrt{a}}\right)-2\sqrt{a+b\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[x]]*Tan[x],x]
```

```
[Out] 2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]]/Sqrt[a]] - 2*Sqrt[a + b*Cos[x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+b\cos(x)}\,\tan(x)\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a+x}}{x}\,dx, x, b\cos(x)\right)\\
&= -2\sqrt{a+b\cos(x)} - a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\cos(x)\right)\\
&= -2\sqrt{a+b\cos(x)} - (2a)\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\cos(x)}\right)\\
&= 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos(x)}}{\sqrt{a}}\right) - 2\sqrt{a+b\cos(x)}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.00

$$2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cos(x)}}{\sqrt{a}}\right) - 2\sqrt{a+b\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[x]]*Tan[x],x]
```

```
[Out] 2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cos[x]]/Sqrt[a]] - 2*Sqrt[a + b*Cos[x]]
```

fricas [A] time = 1.34, size = 109, normalized size = 2.95

$$\left[\frac{1}{2}\sqrt{a}\,\log\left(-\frac{b^2\cos(x)^2 + 8\,ab\cos(x) + 4\,(b\cos(x)+2\,a)\sqrt{b\cos(x)+a}\,\sqrt{a} + 8\,a^2}{\cos(x)^2}\right) - 2\,\sqrt{b\cos(x)+a}\,, -\sqrt{-a}\,\text{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log(-(b^2*cos(x)^2 + 8*a*b*cos(x) + 4*(b*cos(x) + 2*a)*sqrt(b*
cos(x) + a)*sqrt(a) + 8*a^2)/cos(x)^2) - 2*sqrt(b*cos(x) + a), -sqrt(-a)*ar
ctan(2*sqrt(b*cos(x) + a)*sqrt(-a)/(b*cos(x) + 2*a)) - 2*sqrt(b*cos(x) + a)
]
```

giac [A] time = 0.38, size = 34, normalized size = 0.92

$$-\frac{2\,a\arctan\left(\frac{\sqrt{b\cos(x)+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - 2\,\sqrt{b\cos(x)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))^(1/2)*tan(x),x, algorithm="giac")
```

```
[Out] -2*a*arctan(sqrt(b*cos(x) + a)/sqrt(-a))/sqrt(-a) - 2*sqrt(b*cos(x) + a)
```

maple [A] time = 0.04, size = 30, normalized size = 0.81

$$2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\cos(x)}}{\sqrt{a}}\right)\sqrt{a} - 2\sqrt{a+b\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(x))^(1/2)*tan(x),x)
```

```
[Out] 2*arctanh((a+b*cos(x))^(1/2)/a^(1/2))*a^(1/2)-2*(a+b*cos(x))^(1/2)
```

maxima [A] time = 1.67, size = 46, normalized size = 1.24

$$-\sqrt{a}\,\log\left(\frac{\sqrt{b\cos(x)+a}-\sqrt{a}}{\sqrt{b\cos(x)+a}+\sqrt{a}}\right) - 2\,\sqrt{b\cos(x)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] -sqrt(a)*log((sqrt(b*cos(x) + a) - sqrt(a))/(sqrt(b*cos(x) + a) + sqrt(a)))
 - 2*sqrt(b*cos(x) + a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \tan(x)\,\sqrt{a+b\,\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(a + b*cos(x))^(1/2),x)
```

```
[Out] int(tan(x)*(a + b*cos(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\cos{(x)}}\,\tan{(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(x))**(1/2)*tan(x),x)
```

```
[Out] Integral(sqrt(a + b*cos(x))*tan(x), x)
```

3.20 $\int \frac{\tan(x)}{\sqrt{a+b\cos(x)}}\,dx$

Optimal. Leaf size=24

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cos(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] 2*arctanh((a+b*cos(x))^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2721, 63, 207}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cos(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/Sqrt[a + b*Cos[x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Cos[x]]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan(x)}{\sqrt{a+b\cos(x)}}\,dx = -\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\cos(x)\right)$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\cos(x)}\right)\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cos(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\cos(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/Sqrt[a + b*Cos[x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Cos[x]]/Sqrt[a]])/Sqrt[a]
```

fricas [B] time = 0.71, size = 98, normalized size = 4.08

$$\left[\frac{\log\left(\frac{b^2\cos(x)^2+8\,ab\cos(x)+4\,(b\cos(x)+2\,a)\sqrt{b\cos(x)+a}\,\sqrt{a}+8\,a^2}{\cos(x)^2}\right)}{2\,\sqrt{a}}, -\frac{\sqrt{-a}\,\arctan\left(\frac{(b\cos(x)+2\,a)\sqrt{b\cos(x)+a}\,\sqrt{-a}}{2\,\left(ab\cos(x)+a^2\right)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b^2*cos(x)^2 + 8*a*b*cos(x) + 4*(b*cos(x) + 2*a)*sqrt(b*cos(x) +
a)*sqrt(a) + 8*a^2)/cos(x)^2)/sqrt(a), -sqrt(-a)*arctan(1/2*(b*cos(x) + 2*a
)*sqrt(b*cos(x) + a)*sqrt(-a)/(a*b*cos(x) + a^2))/a]
```

giac [A] time = 0.40, size = 22, normalized size = 0.92

$$-\frac{2\,\arctan\left(\frac{\sqrt{b\cos(x)+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x))^(1/2),x, algorithm="giac")
[Out] -2*arctan(sqrt(b*cos(x) + a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.04, size = 19, normalized size = 0.79

$$\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b\cos(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a+b*cos(x))^(1/2),x)
[Out] 2*arctanh((a+b*cos(x))^(1/2)/a^(1/2))/a^(1/2)
```

maxima [A] time = 1.18, size = 35, normalized size = 1.46

$$-\frac{\log\left(\frac{\sqrt{b\cos(x)+a}-\sqrt{a}}{\sqrt{b\cos(x)+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x))^(1/2),x, algorithm="maxima")
[Out] -log((sqrt(b*cos(x) + a) - sqrt(a))/(sqrt(b*cos(x) + a) + sqrt(a)))/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(x)}{\sqrt{a+b\,\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a + b*cos(x))^(1/2),x)
[Out] int(tan(x)/(a + b*cos(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan{\left(x \right)}}{\sqrt{a+b\cos{\left(x \right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a+b*cos(x))**(1/2),x)
[Out] Integral(tan(x)/sqrt(a + b*cos(x)), x)
```

3.21 $\int \frac{\sqrt{e\tan(c+dx)}}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=204

$$\frac{2\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right)\middle|-1\right)}{d\sqrt{b-a}\sqrt{a+b}\sqrt{\sin(c+dx)}}-\frac{2\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\,\Pi\left(-\frac{\sqrt{b}}{\sqrt{a}}\right.}{d\sqrt{b-a}\sqrt{a+b}\sqrt{\sin}}$$

```
[Out] -2*EllipticPi(sin(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/
2),I)*2^(1/2)*cos(d*x+c)^(1/2)*(e*tan(d*x+c))^(1/2)/d/(-a+b)^(1/2)/(a+b)^(1
/2)/sin(d*x+c)^(1/2)+2*EllipticPi(sin(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),(-a
+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*cos(d*x+c)^(1/2)*(e*tan(d*x+c))^(1/2)/d/(-
a+b)^(1/2)/(a+b)^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2733, 2730, 2906, 2905, 490, 1213, 537}

$$\frac{2\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)+1}}\right)\middle|-1\right)}{d\sqrt{b-a}\sqrt{a+b}\sqrt{\sin(c+dx)}}-\frac{2\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\,\Pi\left(-\frac{\sqrt{b}}{\sqrt{a}}\right.}{d\sqrt{b-a}\sqrt{a+b}\sqrt{\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Tan[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*Sqrt[2]*Sqrt[Cos[c + d*x]]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcS
in[Sqrt[Sin[c + d*x]]/Sqrt[1 + Cos[c + d*x]]], -1]*Sqrt[e*Tan[c + d*x]])/(S
qrt[-a + b]*Sqrt[a + b]*d*Sqrt[Sin[c + d*x]]) + (2*Sqrt[2]*Sqrt[Cos[c + d*x
]]*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[Sin[c + d*x]]/Sqrt[1 +
Cos[c + d*x]]], -1]*Sqrt[e*Tan[c + d*x]])/(Sqrt[-a + b]*Sqrt[a + b]*d*Sqrt[
Sin[c + d*x]])
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
```

```
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 2730

```
Int[1/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(g_)*tan[(e_.) + (f_.)*(
x_)]]), x_Symbol] :> Dist[Sqrt[Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*Sqrt[g*Tan
[e + f*x]]), Int[Sqrt[Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x]
)), x], x] /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2733

```
Int[(cot[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[g^(2*IntPart[p])*(g*Cot[e + f*x])^FracPart[p]
*(g*Tan[e + f*x])^FracPart[p], Int[(a + b*Sin[e + f*x])^m/(g*Tan[e + f*x])^
p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 2905

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(-4*Sqrt[2]*g)/f, Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2906

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rubi steps

$$\int \frac{\sqrt{e\tan(c+dx)}}{a+b\cos(c+dx)}\,dx = \left(\sqrt{e\cot(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \frac{1}{(a+b\cos(c+dx))\sqrt{e\cot(c+dx)}}\,dx$$

$$= \frac{\left(\sqrt{-\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \frac{\sqrt{\sin(c+dx)}}{\sqrt{-\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{\sqrt{\sin(c+dx)}}$$

$$= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\int \frac{\sqrt{\sin(c+dx)}}{\sqrt{\cos(c+dx)}\,(a+b\cos(c+dx))}\,dx}{\sqrt{\sin(c+dx)}}$$

$$= \frac{\left(4\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}\left(a+b+(a-b)x^4\right)}\,dx, x, \frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right)}{d\sqrt{\sin(c+dx)}}$$

$$= \frac{\left(2\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{\left(\sqrt{a+b}-\sqrt{-a+b}\,x^2\right)\sqrt{1-x^4}}\,dx, x, \frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right)}{\sqrt{-a+b}\,d\sqrt{\sin(c+dx)}}$$

$$= \frac{\left(2\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{e\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a+b}-\sqrt{-a+b}\,x^2\right)}\,dx, x, \frac{\sqrt{\sin(c+}}{\sqrt{1+\cos(}}\right.}{\sqrt{-a+b}\,d\sqrt{\sin(c+dx)}}$$

$$= -\frac{2\sqrt{2}\sqrt{\cos(c+dx)}\,\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{\sin(c+dx)}}{\sqrt{1+\cos(c+dx)}}\right)\middle|-1\right)\sqrt{e\tan(c+dx)}}{\sqrt{-a+b}\sqrt{a+b}\,d\sqrt{\sin(c+dx)}} + \frac{2\sqrt{2}\sqrt{\mathrm{co}}}{}$$

Mathematica [C] time = 2.78, size = 363, normalized size = 1.78

$$\frac{2\sqrt{e\tan(c+dx)}\left(a\sqrt{\sec^2(c+dx)}+b\right)\left(\frac{b\tan^{\frac{3}{2}}(c+dx)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\tan^2(c+dx),-\frac{a^2\tan^2(c+dx)}{a^2-b^2}\right)}{3\left(b^2-a^2\right)}+\frac{-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{a}\sqrt{\tan(c+dx)}}{\sqrt[4]{a^2-b^2}}\right)+2\tan}{}\right.}{d\sqrt{\tan(c+dx)}\sqrt{\sec^2(c+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Tan[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*(b + a*Sqrt[Sec[c + d*x]^2])*Sqrt[e*Tan[c + d*x]]*((-2*ArcTan[1 - (Sqrt[
2]*Sqrt[a]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*S
qrt[a]*Sqrt[Tan[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - Sqrt[
2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + a*Tan[c + d*x]] - Log[Sqr
t[a^2 - b^2] + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[c + d*x]] + a*Tan
[c + d*x]])/(4*Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF1[3/4, 1/2, 1
```

```
, 7/4, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Tan[c + d*x]^(
3/2))/(3*(-a^2 + b^2))))/(d*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]^2]*Sqrt[
Tan[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\tan(dx+c)}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*tan(d*x + c))/(b*cos(d*x + c) + a), x)
```

maple [B] time = 0.40, size = 546, normalized size = 2.68

$$\sqrt{\frac{-1+\cos(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{e\sin(dx+c)}{\cos(dx+c)}}\,(1+\cos(dx+c))^2\,(-1+\cos(dx+c))\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(d*x+c))^(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] 1/d*((-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+
c))^(1/2)*((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2)*(e*sin(d*x+c)/cos(d*
x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))*((-a^2+b^2)^(1/2)*EllipticPi((
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+(-(a-b)*(a+b))^(1/2)
),1/2*2^(1/2))-(-a^2+b^2)^(1/2)*EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d
*x+c))^(1/2),-(a-b)/(-a+b+(-(a-b)*(a+b))^(1/2)),1/2*2^(1/2))-a*EllipticPi((
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b)/(a-b+(-(a-b)*(a+b))^(1/2)
),1/2*2^(1/2))+EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),(a-b
)/(a-b+(-(a-b)*(a+b))^(1/2)),1/2*2^(1/2))*b-a*EllipticPi(((1-cos(d*x+c)+sin
(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+(-(a-b)*(a+b))^(1/2)),1/2*2^(1/2))+
EllipticPi(((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))^(1/2),-(a-b)/(-a+b+(-(a-b
```

```
)*(a+b))^(1/2)),1/2*2^(1/2))*b)/sin(d*x+c)^3*2^(1/2)/((-a^2+b^2)^(1/2)-a+b)
/((-a^2+b^2)^(1/2)+a-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\tan\left(dx+c\right)}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))^(1/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*tan(d*x + c))/(b*cos(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\tan\left(c+d\,x\right)}}{a+b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*tan(c + d*x))^(1/2)/(a + b*cos(c + d*x)),x)
```

```
[Out] int((e*tan(c + d*x))^(1/2)/(a + b*cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\tan\left(c+dx\right)}}{a+b\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*tan(d*x+c))**(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*tan(c + d*x))/(a + b*cos(c + d*x)), x)
```

3.22 $\int (a+b\cos (e+f x))^m (g\tan (e+f x))^p \, dx$

Optimal. Leaf size=49

$$(g\tan (e+f x))^p (g\cot (e+f x))^p \text{Int}\left((g\cot (e+f x))^{-p} (a+b\cos (e+f x))^m,x\right)$$

```
[Out] (g*cot(f*x+e))^p*(g*tan(f*x+e))^p*Unintegrable((a+b*cos(f*x+e))^m/((g*cot(f
*x+e))^p),x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\cos (e+f x))^m (g\tan (e+f x))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Cos[e + f*x])^m*(g*Tan[e + f*x])^p,x]
```

```
[Out] (g*Cot[e + f*x])^p*(g*Tan[e + f*x])^p*Defer[Int][(a + b*Cos[e + f*x])^m/(g*
Cot[e + f*x])^p, x]
```

Rubi steps

$$\int (a+b\cos (e+f x))^m (g\tan (e+f x))^p \, dx=\left((g\cot (e+f x))^p (g\tan (e+f x))^p\right) \int (a+b\cos (e+f x))^m (g\cot (e+$$

Mathematica [A] time = 2.47, size = 0, normalized size = 0.00

$$\int (a+b\cos (e+f x))^m (g\tan (e+f x))^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Cos[e + f*x])^m*(g*Tan[e + f*x])^p,x]
```

```
[Out] Integrate[(a + b*Cos[e + f*x])^m*(g*Tan[e + f*x])^p, x]
```

fricas [A] time = 3.21, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(fx+e\right)+a\right)^{m} \left(g\tan\left(fx+e\right)\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Simplification assuming g near 0Unable to ch
eck sign: (pi/x/2)>(-pi/x/2)Unable to check sign: (pi/x/2)>(-pi/x/2)Simplif
ication assuming f near 0Simplification assuming x near 0Simplification ass
uming a near 0Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Evaluation time: 0.55Unable to divide, perhaps due to rounding err
or%%%{-268435456,[0,10,0,10,16,0,0,4]%%%}+%%%{1946157056,[0,10,0,10,14,0,0,
6]%%%}+%%%{-5570035712,[0,10,0,10,12,0,0,8]%%%}+%%%{7985954816,[0,10,0,10,1
0,0,0,10]%%%}+%%%{-5972688896,[0,10,0,10,8,0,0,12]%%%}+%%%{2147483648,[0,10
,0,10,6,0,0,14]%%%}+%%%{-268435456,[0,10,0,10,4,0,0,16]%%%}+%%%{805306368,[
0,9,0,10,16,1,0,4]%%%}+%%%{268435456,[0,9,0,10,16,0,1,4]%%%}+%%%{-590558003
2,[0,9,0,10,14,1,0,6]%%%}+%%%{-2550136832,[0,9,0,10,14,0,1,6]%%%}+%%%{17179
869184,[0,9,0,10,12,1,0,8]%%%}+%%%{9529458688,[0,9,0,10,12,0,1,8]%%%}+%%%{-
25232932864,[0,9,0,10,10,1,0,10]%%%}+%%%{-18119393280,[0,9,0,10,10,0,1,10]%
%%}+%%%{19595788288,[0,9,0,10,8,1,0,12]%%%}+%%%{18656264192,[0,9,0,10,8,0,1
,12]%%%}+%%%{-7516192768,[0,9,0,10,6,1,0,14]%%%}+%%%{-9932111872,[0,9,0,10,
6,0,1,14]%%%}+%%%{1073741824,[0,9,0,10,4,1,0,16]%%%}+%%%{2147483648,[0,9,0,
10,4,0,1,16]%%%}+%%%{-872415232,[0,8,0,10,16,2,0,4]%%%}+%%%{-939524096,[0,8
,0,10,16,1,1,4]%%%}+%%%{738197504,[0,8,0,10,16,0,2,4]%%%}+%%%{6241124352,[0
,8,0,10,14,2,0,6]%%%}+%%%{8455716864,[0,8,0,10,14,1,1,6]%%%}+%%%{-523449139
2,[0,8,0,10,14,0,2,6]%%%}+%%%{-18320719872,[0,8,0,10,12,2,0,8]%%%}+%%%{-304
67424256,[0,8,0,10,12,1,1,8]%%%}+%%%{14763950080,[0,8,0,10,12,0,2,8]%%%}+%%
%{27850178560,[0,8,0,10,10,2,0,10]%%%}+%%%{56505663488,[0,8,0,10,10,1,1,10]
%%%}+%%%{-20803747840,[0,8,0,10,10,0,2,10]%%%}+%%%{-22951231488,[0,8,0,10,8
,2,0,12]%%%}+%%%{-57176752128,[0,8,0,10,8,1,1,12]%%%}+%%%{15099494400,[0,8,
0,10,8,0,2,12]%%%}+%%%{9663676416,[0,8,0,10,6,2,0,14]%%%}+%%%{30064771072,[
0,8,0,10,6,1,1,14]%%%}+%%%{-5100273664,[0,8,0,10,6,0,2,14]%%%}+%%%{-1610612
736,[0,8,0,10,4,2,0,16]%%%}+%%%{-6442450944,[0,8,0,10,4,1,1,16]%%%}+%%%{536
870912,[0,8,0,10,4,0,2,16]%%%}+%%%{402653184,[0,7,0,10,16,3,0,4]%%%}+%%%{12
07959552,[0,7,0,10,16,2,1,4]%%%}+%%%{-2013265920,[0,7,0,10,16,1,2,4]%%%}+%%
%{-671088640,[0,7,0,10,16,0,3,4]%%%}+%%%{-2550136832,[0,7,0,10,14,3,0,6]%%%
}+%%%{-10066329600,[0,7,0,10,14,2,1,6]%%%}+%%%{14898167808,[0,7,0,10,14,1,2
```

,6]%%%}+%%%{6845104128,[0,7,0,10,14,0,3,6]%%%}+%%%{7381975040,[0,7,0,10,12,
3,0,8]%%%}+%%%{34225520640,[0,7,0,10,12,2,1,8]%%%}+%%%{-43352326144,[0,7,0,
10,12,1,2,8]%%%}+%%%{-26709327872,[0,7,0,10,12,0,3,8]%%%}+%%%{-11945377792,
[0,7,0,10,10,3,0,10]%%%}+%%%{-60800630784,[0,7,0,10,10,2,1,10]%%%}+%%%{6294
8114432,[0,7,0,10,10,1,2,10]%%%}+%%%{52210696192,[0,7,0,10,10,0,3,10]%%%}+%
%%{11005853696,[0,7,0,10,8,3,0,12]%%%}+%%%{59592671232,[0,7,0,10,8,2,1,12]%
%%}+%%%{-47513075712,[0,7,0,10,8,1,2,12]%%%}+%%%{-54760833024,[0,7,0,10,8,0
,3,12]%%%}+%%%{-5368709120,[0,7,0,10,6,3,0,14]%%%}+%%%{-30601641984,[0,7,0,
10,6,2,1,14]%%%}+%%%{17179869184,[0,7,0,10,6,1,2,14]%%%}+%%%{29527900160,[0
,7,0,10,6,0,3,14]%%%}+%%%{1073741824,[0,7,0,10,4,3,0,16]%%%}+%%%{6442450944
,[0,7,0,10,4,2,1,16]%%%}+%%%{-2147483648,[0,7,0,10,4,1,2,16]%%%}+%%%{-64424
50944,[0,7,0,10,4,0,3,16]%%%}+%%%{-67108864,[0,6,0,10,16,4,0,4]%%%}+%%%{-67
1088640,[0,6,0,10,16,3,1,4]%%%}+%%%{1879048192,[0,6,0,10,16,2,2,4]%%%}+%%%{
2281701376,[0,6,0,10,16,1,3,4]%%%}+%%%{-738197504,[0,6,0,10,16,0,4,4]%%%}+%
%%{268435456,[0,6,0,10,14,4,0,6]%%%}+%%%{4966055936,[0,6,0,10,14,3,1,6]%%%}
+%%%{-14092861440,[0,6,0,10,14,2,2,6]%%%}+%%%{-22145925120,[0,6,0,10,14,1,3
,6]%%%}+%%%{4831838208,[0,6,0,10,14,0,4,6]%%%}+%%%{-671088640,[0,6,0,10,12,
4,0,8]%%%}+%%%{-15166603264,[0,6,0,10,12,3,1,8]%%%}+%%%{42278584320,[0,6,0,
10,12,2,2,8]%%%}+%%%{83886080000,[0,6,0,10,12,1,3,8]%%%}+%%%{-13019119616,[
0,6,0,10,12,0,4,8]%%%}+%%%{1342177280,[0,6,0,10,10,4,0,10]%%%}+%%%{24561844
224,[0,6,0,10,10,3,1,10]%%%}+%%%{-64290291712,[0,6,0,10,10,2,2,10]%%%}+%%%{
-160927055872,[0,6,0,10,10,1,3,10]%%%}+%%%{17716740096,[0,6,0,10,10,0,4,10]
%%%}+%%%{-1677721600,[0,6,0,10,8,4,0,12]%%%}+%%%{-22280142848,[0,6,0,10,8,3
,1,12]%%%}+%%%{51942260736,[0,6,0,10,8,2,2,12]%%%}+%%%{166698418176,[0,6,0,
10,8,1,3,12]%%%}+%%%{-12280922112,[0,6,0,10,8,0,4,12]%%%}+%%%{1073741824,[0
,6,0,10,6,4,0,14]%%%}+%%%{10737418240,[0,6,0,10,6,3,1,14]%%%}+%%%{-20937965
568,[0,6,0,10,6,2,2,14]%%%}+%%%{-89120571392,[0,6,0,10,6,1,3,14]%%%}+%%%{37
58096384,[0,6,0,10,6,0,4,14]%%%}+%%%{-268435456,[0,6,0,10,4,4,0,16]%%%}+%%%
{-2147483648,[0,6,0,10,4,3,1,16]%%%}+%%%{3221225472,[0,6,0,10,4,2,2,16]%%%}
+%%%{19327352832,[0,6,0,10,4,1,3,16]%%%}+%%%{-268435456,[0,6,0,10,4,0,4,16]
%%%}+%%%{134217728,[0,5,0,10,16,4,1,4]%%%}+%%%{-671088640,[0,5,0,10,16,3,2,
4]%%%}+%%%{-2818572288,[0,5,0,10,16,2,3,4]%%%}+%%%{1744830464,[0,5,0,10,16,
1,4,4]%%%}+%%%{536870912,[0,5,0,10,16,0,5,4]%%%}+%%%{-805306368,[0,5,0,10,1
4,4,1,6]%%%}+%%%{4429185024,[0,5,0,10,14,3,2,6]%%%}+%%%{25367150592,[0,5,0,
10,14,2,3,6]%%%}+%%%{-12482248704,[0,5,0,10,14,1,4,6]%%%}+%%%{-6039797760,[
0,5,0,10,14,0,5,6]%%%}+%%%{1879048192,[0,5,0,10,12,4,1,8]%%%}+%%%{-13555990
528,[0,5,0,10,12,3,2,8]%%%}+%%%{-91402272768,[0,5,0,10,12,2,3,8]%%%}+%%%{35
567697920,[0,5,0,10,12,1,4,8]%%%}+%%%{24830279680,[0,5,0,10,12,0,5,8]%%%}+%
%%{-2147483648,[0,5,0,10,10,4,1,10]%%%}+%%%{22951231488,[0,5,0,10,10,3,2,10
]%%%}+%%%{169516990464,[0,5,0,10,10,2,3,10]%%%}+%%%{-50331648000,[0,5,0,10,
10,1,4,10]%%%}+%%%{-50063212544,[0,5,0,10,10,0,5,10]%%%}+%%%{1207959552,[0,
5,0,10,8,4,1,12]%%%}+%%%{-21743271936,[0,5,0,10,8,3,2,12]%%%}+%%%{-17153025
6384,[0,5,0,10,8,2,3,12]%%%}+%%%{36238786560,[0,5,0,10,8,1,4,12]%%%}+%%%{53
552873472,[0,5,0,10,8,0,5,12]%%%}+%%%{-268435456,[0,5,0,10,6,4,1,14]%%%}+%%
%{10737418240,[0,5,0,10,6,3,2,14]%%%}+%%%{90194313216,[0,5,0,10,6,2,3,14]%%

%}+%%%{-11811160064,[0,5,0,10,6,1,4,14]%%%}+%%%{-29259464704,[0,5,0,10,6,0,
5,14]%%%}+%%%{-2147483648,[0,5,0,10,4,3,2,16]%%%}+%%%{-19327352832,[0,5,0,1
0,4,2,3,16]%%%}+%%%{1073741824,[0,5,0,10,4,1,4,16]%%%}+%%%{6442450944,[0,5,
0,10,4,0,5,16]%%%}+%%%{67108864,[0,4,0,10,16,4,2,4]%%%}+%%%{1476395008,[0,4
,0,10,16,3,3,4]%%%}+%%%{-1207959552,[0,4,0,10,16,2,4,4]%%%}+%%%{-1744830464
,[0,4,0,10,16,1,5,4]%%%}+%%%{335544320,[0,4,0,10,16,0,6,4]%%%}+%%%{-1167694
2336,[0,4,0,10,14,3,3,6]%%%}+%%%{9663676416,[0,4,0,10,14,2,4,6]%%%}+%%%{189
24699648,[0,4,0,10,14,1,5,6]%%%}+%%%{-1744830464,[0,4,0,10,14,0,6,6]%%%}+%%
%{-134217728,[0,4,0,10,12,4,2,8]%%%}+%%%{37983617024,[0,4,0,10,12,3,3,8]%%%
}+%%%{-29796335616,[0,4,0,10,12,2,4,8]%%%}+%%%{-76369887232,[0,4,0,10,12,1,
5,8]%%%}+%%%{4026531840,[0,4,0,10,12,0,6,8]%%%}+%%%{-805306368,[0,4,0,10,10
,4,2,10]%%%}+%%%{-65095598080,[0,4,0,10,10,3,3,10]%%%}+%%%{45097156608,[0,4
,0,10,10,2,4,10]%%%}+%%%{152337121280,[0,4,0,10,10,1,5,10]%%%}+%%%{-4966055
936,[0,4,0,10,10,0,6,10]%%%}+%%%{2214592512,[0,4,0,10,8,4,2,12]%%%}+%%%{620
08590336,[0,4,0,10,8,3,3,12]%%%}+%%%{-35030827008,[0,4,0,10,8,2,4,12]%%%}+%
%%{-161866579968,[0,4,0,10,8,1,5,12]%%%}+%%%{3154116608,[0,4,0,10,8,0,6,12]
%%%}+%%%{-1879048192,[0,4,0,10,6,4,2,14]%%%}+%%%{-31138512896,[0,4,0,10,6,3
,3,14]%%%}+%%%{12884901888,[0,4,0,10,6,2,4,14]%%%}+%%%{88046829568,[0,4,0,1
0,6,1,5,14]%%%}+%%%{-805306368,[0,4,0,10,6,0,6,14]%%%}+%%%{536870912,[0,4,0
,10,4,4,2,16]%%%}+%%%{6442450944,[0,4,0,10,4,3,3,16]%%%}+%%%{-1610612736,[0
,4,0,10,4,2,4,16]%%%}+%%%{-19327352832,[0,4,0,10,4,1,5,16]%%%}+%%%{-2684354
56,[0,3,0,10,16,4,3,4]%%%}+%%%{134217728,[0,3,0,10,16,3,4,4]%%%}+%%%{201326
5920,[0,3,0,10,16,2,5,4]%%%}+%%%{-671088640,[0,3,0,10,16,1,6,4]%%%}+%%%{-13
4217728,[0,3,0,10,16,0,7,4]%%%}+%%%{1610612736,[0,3,0,10,14,4,3,6]%%%}+%%%{
-1207959552,[0,3,0,10,14,3,4,6]%%%}+%%%{-20535312384,[0,3,0,10,14,2,5,6]%%%
}+%%%{3892314112,[0,3,0,10,14,1,6,6]%%%}+%%%{1744830464,[0,3,0,10,14,0,7,6]
%%%}+%%%{-3758096384,[0,3,0,10,12,4,3,8]%%%}+%%%{4966055936,[0,3,0,10,12,3,
4,8]%%%}+%%%{80127983616,[0,3,0,10,12,2,5,8]%%%}+%%%{-9797894144,[0,3,0,10,
12,1,6,8]%%%}+%%%{-7650410496,[0,3,0,10,12,0,7,8]%%%}+%%%{4294967296,[0,3,0
,10,10,4,3,10]%%%}+%%%{-10066329600,[0,3,0,10,10,3,4,10]%%%}+%%%{-156632088
576,[0,3,0,10,10,2,5,10]%%%}+%%%{12750684160,[0,3,0,10,10,1,6,10]%%%}+%%%{1
5971909632,[0,3,0,10,10,0,7,10]%%%}+%%%{-2415919104,[0,3,0,10,8,4,3,12]%%%}
+%%%{10468982784,[0,3,0,10,8,3,4,12]%%%}+%%%{164282499072,[0,3,0,10,8,2,5,1
2]%%%}+%%%{-8321499136,[0,3,0,10,8,1,6,12]%%%}+%%%{-17448304640,[0,3,0,10,8
,0,7,12]%%%}+%%%{536870912,[0,3,0,10,6,4,3,14]%%%}+%%%{-5368709120,[0,3,0,1
0,6,3,4,14]%%%}+%%%{-88583700480,[0,3,0,10,6,2,5,14]%%%}+%%%{2147483648,[0,
3,0,10,6,1,6,14]%%%}+%%%{9663676416,[0,3,0,10,6,0,7,14]%%%}+%%%{1073741824,
[0,3,0,10,4,3,4,16]%%%}+%%%{19327352832,[0,3,0,10,4,2,5,16]%%%}+%%%{-214748
3648,[0,3,0,10,4,0,7,16]%%%}+%%%{67108864,[0,2,0,10,16,4,4,4]%%%}+%%%{-9395
24096,[0,2,0,10,16,3,5,4]%%%}+%%%{268435456,[0,2,0,10,16,2,6,4]%%%}+%%%{402
653184,[0,2,0,10,16,1,7,4]%%%}+%%%{-67108864,[0,2,0,10,16,0,8,4]%%%}+%%%{-8
05306368,[0,2,0,10,14,4,4,6]%%%}+%%%{8455716864,[0,2,0,10,14,3,5,6]%%%}+%%%
{-2013265920,[0,2,0,10,14,2,6,6]%%%}+%%%{-5234491392,[0,2,0,10,14,1,7,6]%%%
}+%%%{201326592,[0,2,0,10,14,0,8,6]%%%}+%%%{2281701376,[0,2,0,10,12,4,4,8]%
%%}+%%%{-30467424256,[0,2,0,10,12,3,5,8]%%%}+%%%{6039797760,[0,2,0,10,12,2,

6,8]%%%}+%%%{22951231488,[0,2,0,10,12,1,7,8]%%%}+%%%{-201326592,[0,2,0,10,1
2,0,8,8]%%%}+%%%{-2415919104,[0,2,0,10,10,4,4,10]%%%}+%%%{56505663488,[0,2,
0,10,10,3,5,10]%%%}+%%%{-8724152320,[0,2,0,10,10,2,6,10]%%%}+%%%{-479157288
96,[0,2,0,10,10,1,7,10]%%%}+%%%{67108864,[0,2,0,10,10,0,8,10]%%%}+%%%{60397
9776,[0,2,0,10,8,4,4,12]%%%}+%%%{-57176752128,[0,2,0,10,8,3,5,12]%%%}+%%%{6
039797760,[0,2,0,10,8,2,6,12]%%%}+%%%{52344913920,[0,2,0,10,8,1,7,12]%%%}+%
%%{536870912,[0,2,0,10,6,4,4,14]%%%}+%%%{30064771072,[0,2,0,10,6,3,5,14]%%%
}+%%%{-1610612736,[0,2,0,10,6,2,6,14]%%%}+%%%{-28991029248,[0,2,0,10,6,1,7,
14]%%%}+%%%{-268435456,[0,2,0,10,4,4,4,16]%%%}+%%%{-6442450944,[0,2,0,10,4,
3,5,16]%%%}+%%%{6442450944,[0,2,0,10,4,1,7,16]%%%}+%%%{134217728,[0,1,0,10,
16,4,5,4]%%%}+%%%{134217728,[0,1,0,10,16,3,6,4]%%%}+%%%{-402653184,[0,1,0,1
0,16,2,7,4]%%%}+%%%{134217728,[0,1,0,10,16,1,8,4]%%%}+%%%{-805306368,[0,1,0
,10,14,4,5,6]%%%}+%%%{-671088640,[0,1,0,10,14,3,6,6]%%%}+%%%{5234491392,[0,
1,0,10,14,2,7,6]%%%}+%%%{-402653184,[0,1,0,10,14,1,8,6]%%%}+%%%{1879048192,
[0,1,0,10,12,4,5,8]%%%}+%%%{1207959552,[0,1,0,10,12,3,6,8]%%%}+%%%{-2295123
1488,[0,1,0,10,12,2,7,8]%%%}+%%%{402653184,[0,1,0,10,12,1,8,8]%%%}+%%%{-214
7483648,[0,1,0,10,10,4,5,10]%%%}+%%%{-939524096,[0,1,0,10,10,3,6,10]%%%}+%%
%{47915728896,[0,1,0,10,10,2,7,10]%%%}+%%%{-134217728,[0,1,0,10,10,1,8,10]%
%%}+%%%{1207959552,[0,1,0,10,8,4,5,12]%%%}+%%%{268435456,[0,1,0,10,8,3,6,12
]%%%}+%%%{-52344913920,[0,1,0,10,8,2,7,12]%%%}+%%%{-268435456,[0,1,0,10,6,4
,5,14]%%%}+%%%{28991029248,[0,1,0,10,6,2,7,14]%%%}+%%%{-6442450944,[0,1,0,1
0,4,2,7,16]%%%}+%%%{-67108864,[0,0,0,10,16,4,6,4]%%%}+%%%{134217728,[0,0,0,
10,16,3,7,4]%%%}+%%%{-67108864,[0,0,0,10,16,2,8,4]%%%}+%%%{536870912,[0,0,0
,10,14,4,6,6]%%%}+%%%{-1744830464,[0,0,0,10,14,3,7,6]%%%}+%%%{201326592,[0,
0,0,10,14,2,8,6]%%%}+%%%{-1476395008,[0,0,0,10,12,4,6,8]%%%}+%%%{7650410496
,[0,0,0,10,12,3,7,8]%%%}+%%%{-201326592,[0,0,0,10,12,2,8,8]%%%}+%%%{1879048
192,[0,0,0,10,10,4,6,10]%%%}+%%%{-15971909632,[0,0,0,10,10,3,7,10]%%%}+%%%{
67108864,[0,0,0,10,10,2,8,10]%%%}+%%%{-1140850688,[0,0,0,10,8,4,6,12]%%%}+%
%%{17448304640,[0,0,0,10,8,3,7,12]%%%}+%%%{268435456,[0,0,0,10,6,4,6,14]%%%
}+%%%{-9663676416,[0,0,0,10,6,3,7,14]%%%}+%%%{2147483648,[0,0,0,10,4,3,7,16
]%%%} / %%%{1024,[0,4,0,4,8,0,0,0]%%%}+%%%{-4352,[0,4,0,4,6,0,0,2]%%%}+%%%{
5120,[0,4,0,4,4,0,0,4]%%%}+%%%{-1024,[0,4,0,4,2,0,0,6]%%%}+%%%{-1024,[0,3,0
,4,8,1,0,0]%%%}+%%%{-1024,[0,3,0,4,8,0,1,0]%%%}+%%%{4608,[0,3,0,4,6,1,0,2]%
%%}+%%%{6656,[0,3,0,4,6,0,1,2]%%%}+%%%{-6144,[0,3,0,4,4,1,0,4]%%%}+%%%{-133
12,[0,3,0,4,4,0,1,4]%%%}+%%%{2048,[0,3,0,4,2,1,0,6]%%%}+%%%{8192,[0,3,0,4,2
,0,1,6]%%%}+%%%{256,[0,2,0,4,8,2,0,0]%%%}+%%%{1536,[0,2,0,4,8,1,1,0]%%%}+%%
%{-768,[0,2,0,4,8,0,2,0]%%%}+%%%{-256,[0,2,0,4,6,2,0,2]%%%}+%%%{-8192,[0,2,
0,4,6,1,1,2]%%%}+%%%{2816,[0,2,0,4,6,0,2,2]%%%}+%%%{1024,[0,2,0,4,4,2,0,4]%
%%}+%%%{14336,[0,2,0,4,4,1,1,4]%%%}+%%%{-3072,[0,2,0,4,4,0,2,4]%%%}+%%%{-10
24,[0,2,0,4,2,2,0,6]%%%}+%%%{-8192,[0,2,0,4,2,1,1,6]%%%}+%%%{-512,[0,1,0,4,
8,2,1,0]%%%}+%%%{512,[0,1,0,4,8,0,3,0]%%%}+%%%{1536,[0,1,0,4,6,2,1,2]%%%}+%
%%{-1536,[0,1,0,4,6,1,2,2]%%%}+%%%{-5120,[0,1,0,4,6,0,3,2]%%%}+%%%{-1024,[0
,1,0,4,4,2,1,4]%%%}+%%%{2048,[0,1,0,4,4,1,2,4]%%%}+%%%{12288,[0,1,0,4,4,0,3
,4]%%%}+%%%{-8192,[0,1,0,4,2,0,3,6]%%%}+%%%{256,[0,0,0,4,8,2,2,0]%%%}+%%%{-
512,[0,0,0,4,8,1,3,0]%%%}+%%%{256,[0,0,0,4,8,0,4,0]%%%}+%%%{-1280,[0,0,0,4,

```
6,2,2,2]%%%}+%%%{5120,[0,0,0,4,6,1,3,2]%%%}+%%%{1024,[0,0,0,4,4,2,2,4]%%%}+
%%%{-12288,[0,0,0,4,4,1,3,4]%%%}+%%%{8192,[0,0,0,4,2,1,3,6]%%%} Error: Bad
Argument Value
```

maple [A] time = 1.33, size = 0, normalized size = 0.00

$$\int \left(a+b\cos\left(fx+e\right)\right)^{m} \left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^m*(g*tan(f*x+e))^p,x)
[Out] int((a+b*cos(f*x+e))^m*(g*tan(f*x+e))^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(fx+e\right)+a\right)^{m} \left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="maxima")
[Out] integrate((b*cos(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(g\,\tan\left(e+f\,x\right)\right)}^p\,{\left(a+b\,\cos\left(e+f\,x\right)\right)}^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(e + f*x))^p*(a + b*cos(e + f*x))^m,x)
[Out] int((g*tan(e + f*x))^p*(a + b*cos(e + f*x))^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g\tan\left(e+fx\right)\right)^{p} \left(a+b\cos\left(e+fx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**m*(g*tan(f*x+e))**p,x)
[Out] Integral((g*tan(e + f*x))**p*(a + b*cos(e + f*x))**m, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
     as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
   ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
   func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
   expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
   is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
   expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
   leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```